Competitive Trade: Factor Proportion Theory

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(Permanently) Work in Progress

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Introduction

Some Restrictive Features of Ricardian Models:

- Ricardian Theory abstracts from cross-country differences in factor endowment and cross-industry differences in the factor intensity.
  - If there are many factors, every activity in a country uses them in the same proportions so that they can be aggregated into a single composite of the factors.)
  - With multiple factors being aggregated into the composite factor, the cross-country differences in the factor proportions would have no effect on the patterns of trade. The factor endowments merely affect the relative factor prices.
  - International trade would have no effect on the factor prices. No implications for the distribution of income across the owners of different factors. (With non-homothetic preferences, trade has some distributional effects through goods price changes, but not through factor price changes.)

- Extreme Implications on the market share
  - Country A's market share in country B in a particular market is either 0% or 100%, except the marginal sector.
  - A small change in the production cost has a drastic, abrupt, effect in each sector.

- One could also say that Ricardian Theory is highly unsatisfactory as a theory of trade patterns.
  - The question of trade patterns is about "what kinds of countries export (or import) what kinds of goods?"
  - Of course, countries and goods differ in so many dimensions.
  - What we hope from a theory of trade patterns is that it identifies a dimension in which countries differ, so that we can rank them, and a dimension in which goods differ, so that we can rank them, in such a way that we can find some correlations between the ranking of countries and the ranking of goods.
  - Ricardian theory assumed that such a correlation exists, but is silent about where this correlation comes from, and about what are important dimensions in which the countries and goods differ.

Factor Proportion Theory proposes

- Factor Endowment Ratio is an important dimension in which countries differ

- Factor-Intensity is an important dimension in which goods differ.

Two Major models of Factor Proportion Theory:

1. The Ricardo-Viner (Specific Factors) model

2. The Hechscher-Ohlin model and its generalization.

- To focus on the role of factor proportion differences, many studies abstract away from other sources of differences. In particular, they typically assume that the technologies are identical everywhere.

- The assumption of identical technologies may be useful for separating the role of factor endowments from the Ricardian effects. Yet,

- Personally, I feel that too much emphasis has been made on the results that critically hinge on this assumption (such as Factor Price Equalization).

- Later in this part, we will discuss some hybrid models that look at the implications of factor proportion models with technological heterogeneities.

Ricardo-Viner (Specific Factors) Model:

Instead of assuming that the output is a *linear* function of the labor input, as $x_j = L_j/a_j$, **the Ricardo-Viner model** assumes that it is (strictly) *concave* (i.e. *Diminishing Returns*):

$$x_j = F^j(L_j) \qquad \text{with } F_L^j > 0,\ F_{LL}^j < 0, \qquad (j = 1, 2, \ldots, N)$$

Diminishing returns (DR) imply that each sector earns some profits or rents equal to

$$\pi_j \equiv Max_{L_j}\{p_j F^j(L_j) - wL_j\} > 0 \qquad (j = 1, 2, \ldots, N)$$

Alternatively, one may interpret that DR in the Ricardo-Viner model is due to the presence of some hidden factors that are in fixed supply:

$$x_j = F^j(L_j, K_j); \qquad F_L^j > 0,\ F_{LL}^j < 0,\ F_K^j > 0,\ F_{KK}^j < 0,\ F_{KK}^j F_{LL}^j = F_{LK}^j F_{KL}^j$$

where F^j satisfies CRS and K_j is the composite of some hidden factors used *only* in sector-*j*. Then, π_j is the total profit (rent) earned by the hidden factor, K_j, and ρ_j is the profit (rental) rate of the hidden factor, K_j.

$$\pi_j \equiv Max_{L_j}\{p_j F^j(L_j, K_j) - wL_j\} \equiv \rho_j K_j$$

Or, by defining $k_j \equiv K_j / L_j$ & $f_j(k_j) \equiv F^j(L_j, K_j)/L_j = F^j(1, k_j)$,

$$\rho_j = p_j f_j'(k_j) \qquad \& \qquad w = p_j\{f_j'(k_j) - k_j f_j'(k_j)\}.$$

The N-goods **Ricardo-Viner model** can be viewed as the *N*-goods **Specific Factors Model**, a special case of the *N*-goods, (*N*+1)-factors, model with CRS, with

- One *Generic* (or *Mobile*) factor, which can be used in all sectors: often called *Labor*.

- *N Specific* (or *Immobile*) factors, each of which can be used only in one sector: often called *Capital*.

Note*:* A factor is *mobile* when it can be reallocated to different uses. We may be able to think of buildings and land as mobile. Sure, they cannot "move" physically, but different industries can "move in" to the same building or the same location.

Various Interpretations are possible:

- Many machines are specific to a particular industry; labor is not.

- Land (capital) is mostly used in agriculture (manufacturing), so we may treat land (capital) as a specific factor in agriculture (manufacturing). Labor moves between the two sectors. Quite common in development economics.

- Within the agricultural sector, different produces are grown in different locations. Land cannot move to different locations, but agricultural workers can.

- Some human capitals (such as doctors and engineers) are specific to a particular industry, while some human capitals (some administrators) are more mobile.

- Oil and other natural resource deposits may be viewed as a specific factor in the oil (or natural resource) extraction sector.

- When goods produced in different dates are viewed as different goods, land or labor services available in each date may be viewed as specific, while certain storable resource may be viewed as mobile.

etc.

However, following the convention, I will often call the single generic factor, "mobile labor," denoted by L, and the N-specific factors, "specific capital," denoted by K_j.

Factor Endowment Vector: $V = (L; K^T)^T = (L, K_1, K_2, \ldots K_N)^T$

Revenue Function: $R(p, L, K) = Max_{L_1, L_2, \ldots, L_N} \sum_{j=1}^{N} p_j F^j(L_j, K_j) + w\left\{L - \sum_{j=1}^{N} L_j\right\}$

→ F.O.C. $\quad p_j F_L^j(L_j, K_j) = p_j F_L^j\left(L_j / K_j, 1\right) = w = R_L$ for *all j*

$$L = \sum_{j=1}^{N} L_j .$$

- Marginal values of labor are equalized across sectors at the equilibrium wage rate:
- Total labor demand must be equal to total labor supply.

Define $L_j / K_j \equiv \phi^j\left(w / p_j\right)$ by $F_L^j\left(L_j / K_j, 1\right) = w / p_j$. Then,

Labor Market Equilibrium: $L = \sum_{j=1}^{N} L_j = \sum_{j=1}^{N} K_j \left(\dfrac{L_j}{K_j}\right) = \sum_{j=1}^{N} K_j \phi^j\left(\dfrac{w}{p_j}\right).$

For each L, $K = (K_1, K_2, \ldots, K_N)$, and $p = (p_1, p_2, \ldots, p_N)$, this equation determines w, hence, $L_1, L_2, \ldots, L_N$, hence $x = (x_1, x_2, \ldots, x_N)$, hence $\pi_1, \pi_2, \ldots, \pi_N$,

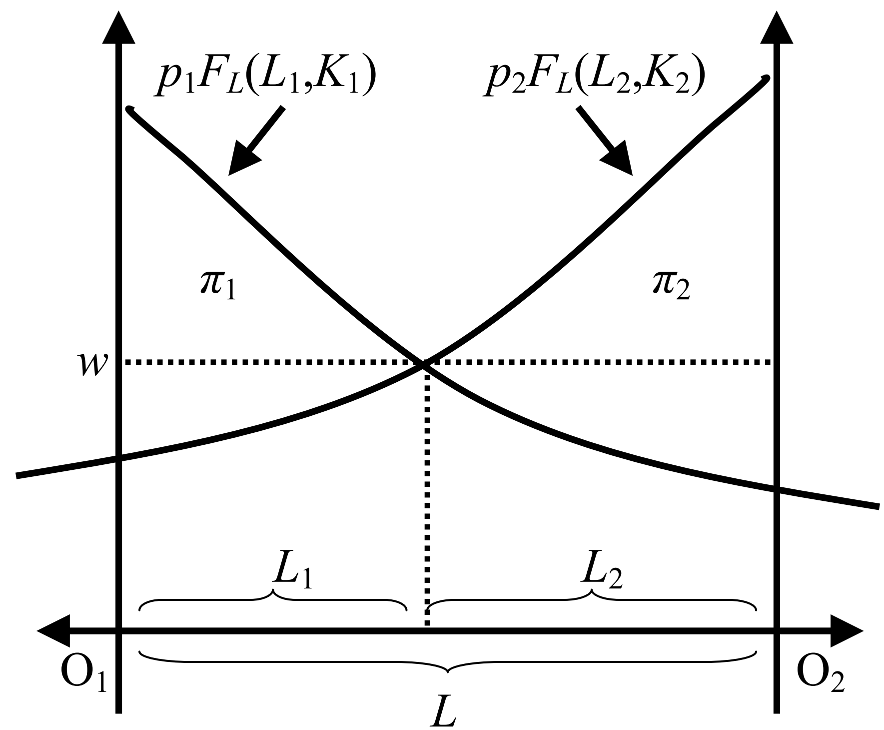
A Graphical Representation of the Labor Market Equilibrium (Two-Sector Case)

- Downward-sloping curve: Marginal Value of Labor (MVL) in Sector 1, measuring L_1 horizontally from the origin, O_1 to the *right*.

- Upward-sloping curve: MVL in Sector 2, measuring L_2 horizontally from the origin, O_2 to the *left*.

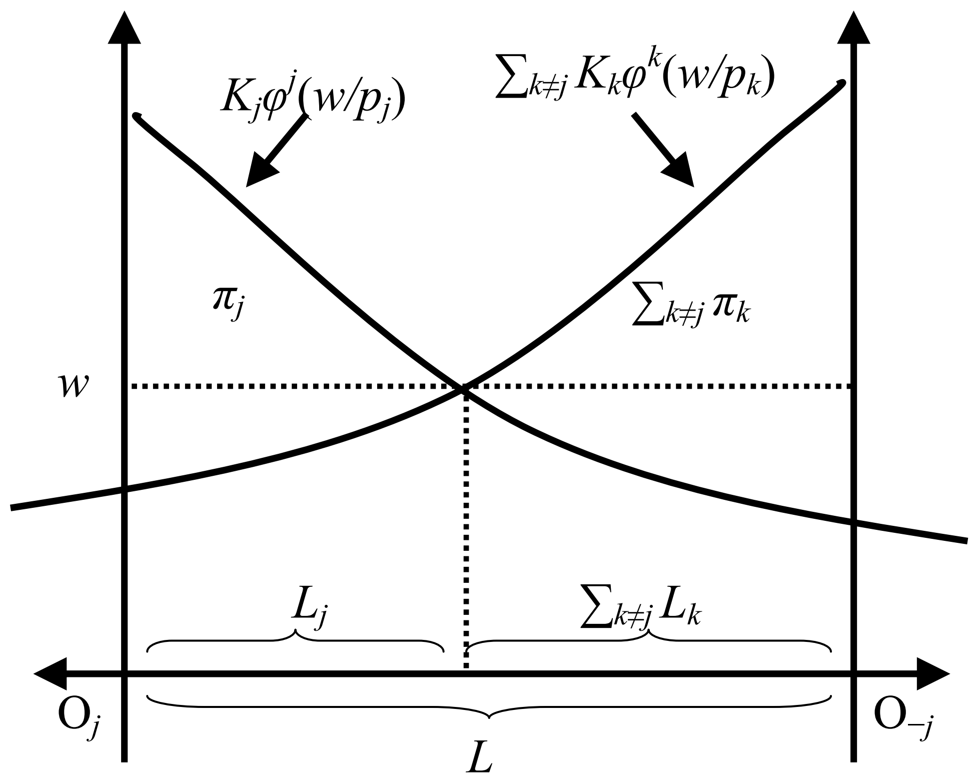
- With the distance between the two origins equal to L, the intersection of the two curves determines the equilibrium wage and the equilibrium allocation of labor.

- The triangular area below the MVL curve for Sector 1 (2) and above the wage line is equal to π_1 (π_2), the total rents in Sector 1 (2).



A Graphical Representation of the Labor Market Equilibrium (*N*-Sector Case)

Different sectors interact only in the single mobile factor market. Hence, to analyze each sector, we can add up the labor demand of all other sectors in the economy.



In other words, a general *N*-sector case essentially preserves the two-sector character.

Some Comparative Statics (with exogenous p):

From Part 1, recall that $x_j = \partial R/\partial p_j$, $\rho_j = \partial R/\partial K_j$, and $w = \partial R/\partial L$.

Effects of Price Changes (with constant Factor Supplies):
Here, let us suppress L and K_j: subscript j means a derivative with respect to p_j. Also, let $\hat{x} \equiv \partial x/x$ denote the rate of change in variable x.

1) $$\frac{\partial x_j}{\partial p_j} = \frac{\partial R_j}{\partial p_j}(p) = R_{jj}(p) > 0 \text{ for all } j;$$

Note: This is expected from the convexity of $R(p,V)$, if $R(p,V)$ is twice differentiable.

2) $$\frac{\partial x_j}{\partial p_i} = R_{ji}(p) = R_{ij}(p) = \frac{\partial x_i}{\partial p_j} < 0 \text{ for all } i \neq j.$$

Note: This is expected for any two-sector model, because, when $N = 2$, $pR_{pp} = 0$ can be written as $p_1R_{11}+p_2R_{12} = p_1R_{21}+p_2R_{22} = 0$, so that $R_{11} > 0$ and $R_{22} > 0$ implies $R_{12} = R_{21} < 0$. This is because, in a two-sector model, an expansion of one sector must always come at the expense of the other sector. What is special about the Specific Factors Model is that this is true for *any N,* because all sectors compete for a single mobile factor.

3) $\hat{p}_j > \hat{p}_i$ for $i \neq j$ $\Rightarrow$ $\hat{\rho}_j > \hat{p}_j > \hat{w} > \hat{p}_i > \hat{\rho}_i$.

Note: Suppose that only the price of good *j* goes up. This makes the owner of the specific factor *j* better off, while making the owner of all other specific factors worse off. The welfare effect on workers is ambiguous. (If the workers spend most of their wage income on good *j,* they could be worse off. Ruffin and Jones (1977) argues that they are likely to be worse off.)

Exercise:

Demonstrate 1)-3) graphically for $N = 2$.

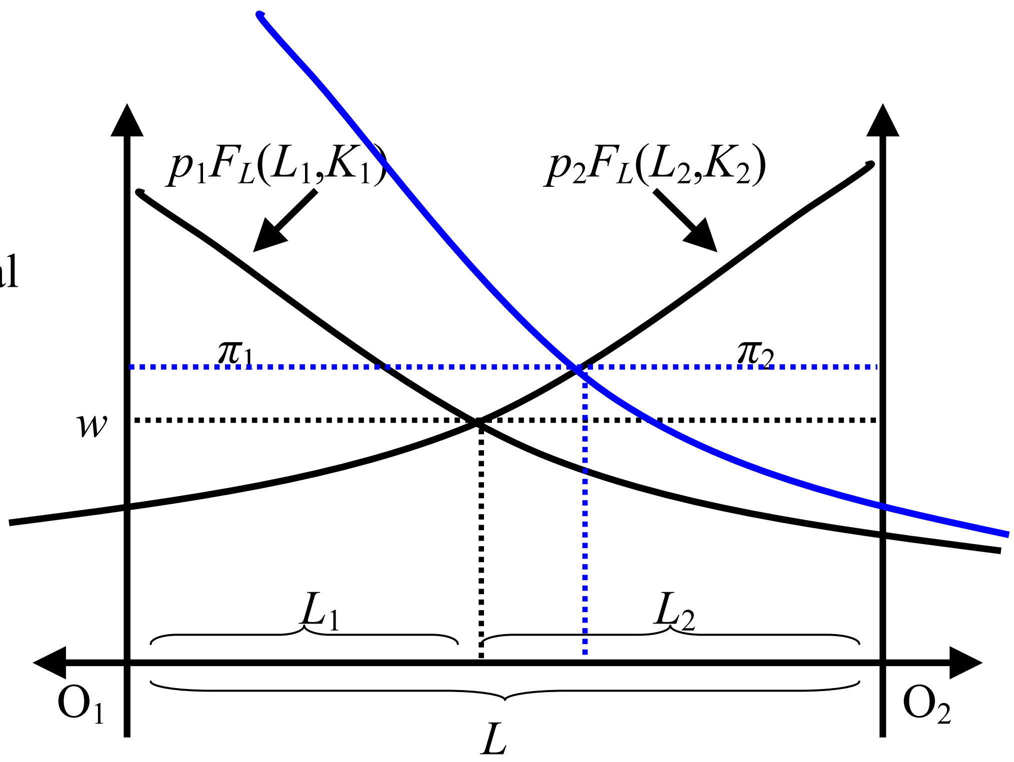
Hint:

Make use of the fact that the *vertical* distance of the MVL curve is proportional to the price. (Thus, a higher price shifts the MVL curve *upward.*)

Exercise:

Prove 1)-3) analytically for $N > 2$.

See Dixit and Norman (1980; pp.38-43).



Effects of Changes in the Generic Factor:

4) $\dfrac{\partial x_j}{\partial L} = R_{p_j L} = \dfrac{\partial w}{\partial p_j} > 0$ and $0 < \dfrac{\partial L_j}{\partial L} = \dfrac{p_j}{w}\dfrac{\partial w}{\partial p_j} < 1$ for all j.

5) $\dfrac{\partial w}{\partial L} = R_{LL} < 0$.

6) $\dfrac{\partial \rho_j}{\partial L} = R_{K_j L} = \dfrac{\partial w}{\partial K_j} > 0$ for all j.

Note: An increase in the generic factor, by making the generic factor cheaper, allows all sectors to expand.

Exercise: Demonstrate 4)-6) graphically for $N = 2$.

Exercise: Prove 4)-6) analytically for $N > 2$.

See Dixit and Norman (1980; pp.38-43).

Effects of Changes in the Specific Factors:

7) $\dfrac{\partial x_j}{\partial K_j} = R_{p_j K_j} = \dfrac{\partial \rho_j}{\partial p_j} > \dfrac{\rho_j}{p_j} = F_K^j > 0$ for all j.

Note: An increase in the specific factor-*j* makes sector-*j* expand *more than* its marginal productivity, because sector-*j* also increases the employment of the generic factor.

8) $\dfrac{\partial w}{\partial K_j} = R_{LK_j} = \dfrac{\partial \rho_j}{\partial L} > 0$ for all j.

9) $\dfrac{\partial x_j}{\partial K_i} = R_{p_j K_i} = \dfrac{\partial \rho_i}{\partial p_j} < 0$ for all i≠j.

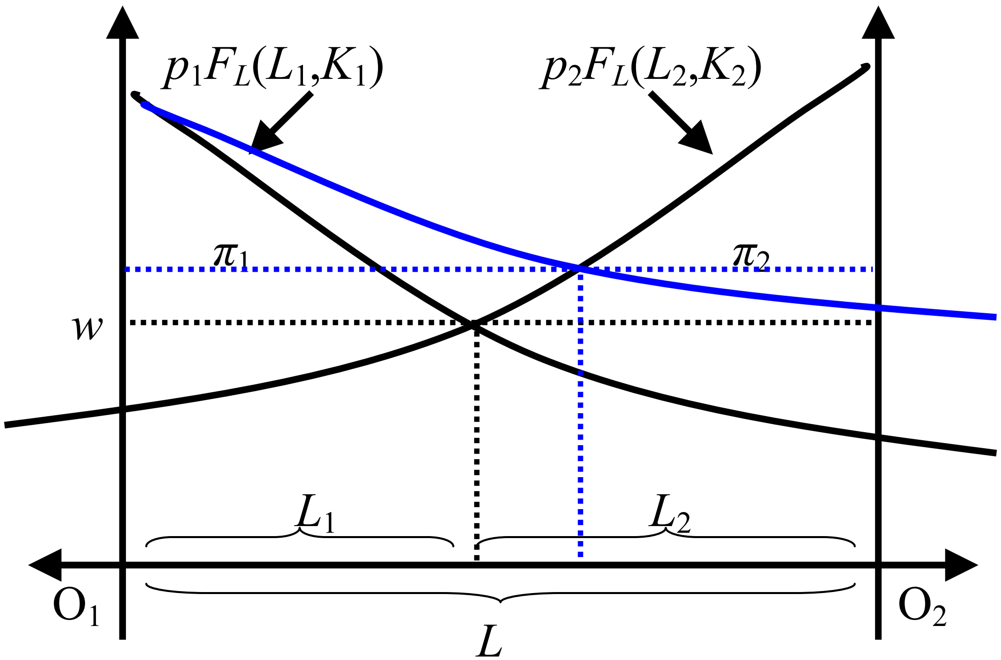
10) $\dfrac{1}{K_j}\dfrac{\partial \pi_j}{\partial K_i} = \dfrac{\partial \rho_j}{\partial K_i} = R_{K_j K_i} = \dfrac{\partial \rho_i}{\partial K_j} = \dfrac{1}{K_i}\dfrac{\partial \pi_i}{\partial K_j} < 0$ for all $i \neq j$.

Note: An increase in the specific factor-*j* squeezes all other sectors because sector-*j*'s higher demand for the generic factor makes the generic factor more expensive. This shows how an increase in a specific factor leads to *unbalanced or biased* growth.

11) $\dfrac{\partial \rho_j}{\partial K_j} = R_{K_j K_j} < 0$ for all j.

12): $\dfrac{\partial \pi_j}{\partial K_j} > (<) 0$ if labor demand by the other sectors is sufficient elastic (inelastic).

Exercise:
Demonstrate 7)-12) graphically for N = 2.
Hint:
Make use of the fact that the *horizontal* distance of the MVL curve is proportional to the endowment of the specific factor. (Thus, a higher K_1 shifts the MVL curve of sector 1 to the *right*.)



Exercise:
Prove 7)-12) analytically for $N > 2$.

See Dixit and Norman (1980; pp.38-43).

Two-Country World Economy:

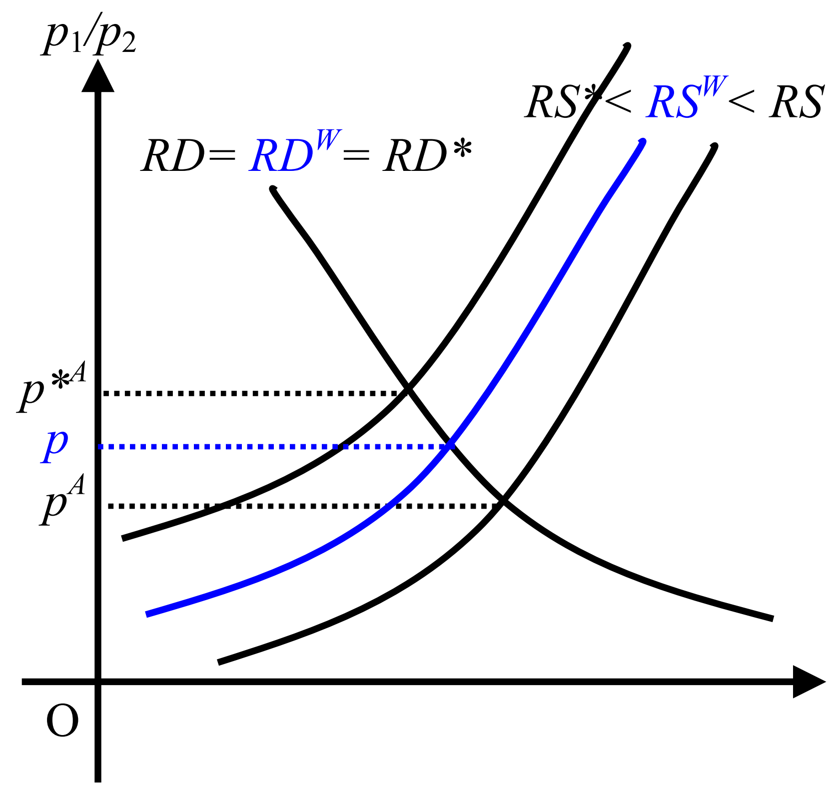
Consider the two-sector specific factor model with the two countries, Home and Foreign, which differ *only* in the factor endowments.

If $\frac{K_1}{L} > \frac{K_1^*}{L^*}$ and $\frac{K_2}{L} = \frac{K_2^*}{L^*}$,

$$\Rightarrow \quad \frac{x_1(p,V)}{x_2(p,V)} > \frac{x_1^*(p,V^*)}{x_2^*(p,V^*)} \text{ for all } p.$$

With the identical homothetic preferences, the two countries share the same Relative Demand curve. Hence,

$$\Rightarrow \quad \left(\frac{p_1}{p_2}\right)^A < \frac{p_1}{p_2} < \left(\frac{p_1^*}{p_2^*}\right)^A.$$



Patterns of Trade:

Home exports Good 1 to Foreign; Foreign exports Good 2 to Home.

Note: The identical technology assumption here is not so restrictive as it seems, since the specific factors endowment differences may be viewed as a proxy for different technologies.

Relative Price Changes:

At Home, p_1/p_2 goes up after trade: $\hat{p}_1 > \hat{p}_2 \Rightarrow \hat{\rho}_1 > \hat{p}_1 > \hat{w} > \hat{p}_2 > \hat{\rho}_2$.
At Foreign, $p^*{}_1/p^*{}_2$ goes down after trade: $\hat{p}_1^* < \hat{p}_2^* \Rightarrow \hat{\rho}_1^* < \hat{p}_1^* < \hat{w}^* < \hat{p}_2^* > \hat{\rho}_2^*$.

In both countries, the owners of the factors specific to the export sector gain, while the owners of the factors specific to the import-competing sectors lose.

Note: When the two countries differ in the endowment of the mobile factor, the prediction is ambiguous, and it depends on the detail of the technologies, particularly the elasticity of substitution between the mobile and specific factor in each sector. See Dixit and Norman (1980; pp.86-87).

Some Advantages of Specific Factors (Ricardo-Viner) Model:

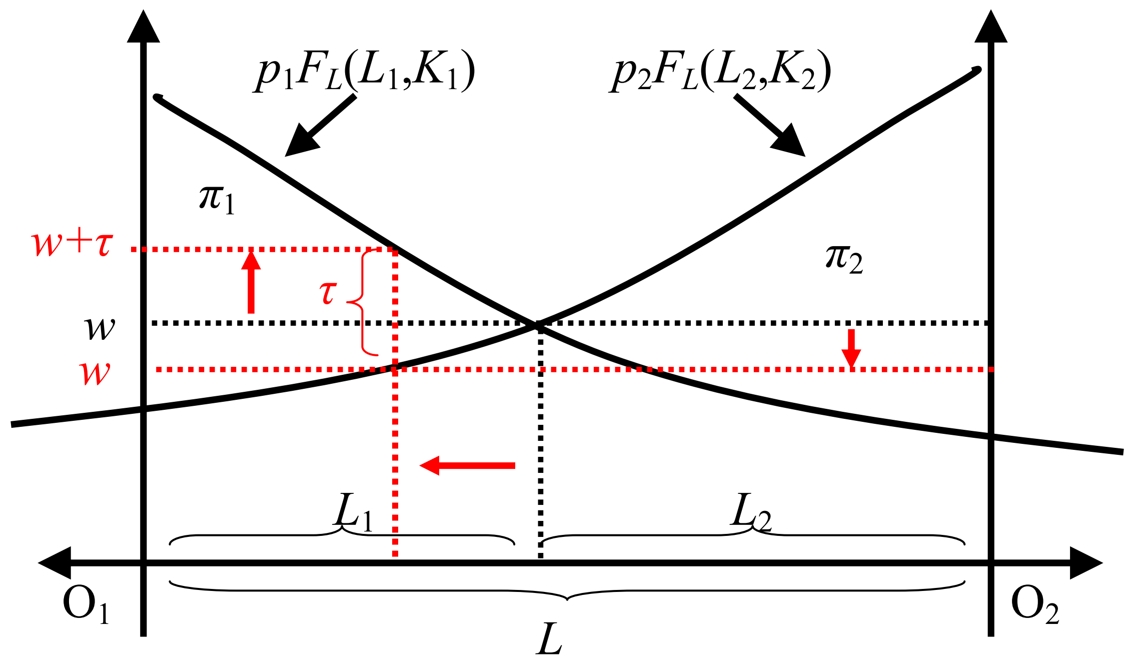
- Many properties of the model do not depend on N.
- Trade does not eliminate sectors producing import goods; they merely shrink.
- Useful framework within which to address political economy of trade policies
- Many comparative statics are intuitive.

Example:

Labor Tax on Sector 1

- increases (reduces) the wage cost in sector 1 (2).
- reduces (increases) the employment and the profit in sector 1 (2).

(The same exercise generates some counter-intuitive results in the Heckscher-Ohlin model.)



An Application: National Resource Abundance and Industrial Development

The Question: What is the effect of having abundant natural sources or highly productive agricultural sectors on industrial development? Consider the following example adopted from Matsuyama (1992).

Two Sectors: Manufacturing and Agriculture, with diminishing returns in labor, as in the Ricardo-Viner Model (or with some specific factors and one mobile factor, labor)

$$X_M = F(n); \qquad F(0) = 0; F' > 0; F'' < 0$$
$$X_A = AG(1-n); \qquad G(0) = 0; G' > 0; G'' < 0.$$

where n is the employment share of the M-sector. The parameter, A, is agricultural productivity, but it may also be viewed as the supply of the factor specific to this sector.

Labor Market Equilibrium: given $p = p_M / p_A$, the equilibrium wage rate, w, satisfies

$$AG'(1-n) = w = pF'(n).$$

Stone-Geary Preferences: $U = \beta\log(C_A - \gamma) + \log(C_M)$, $\gamma > 0$.

F.O.C. $\quad C_A = \gamma + \beta p C_M$.

The Case of a Small Open Economy, which takes p as exogenously given.

Labor Market Condition, $AG'(1-n) = pF'(n)$, alone pins down *n*, the employment share of the M-sector. Hence, *n* is strictly *decreasing* in *A*.

The Case of a Closed Economy; p is endogenously determined to clear

Goods Market Equilibriums: $C_M = X_M = F(n)$; $\quad C_A = X_A = AG(1-n)$

From $C_A = \gamma + \beta p C_M$: $\quad \dfrac{\gamma}{A} = G(1-n) - \beta\dfrac{pF(n)}{A}$.

From $AG'(1-n) = pF'(n)$: $\quad \dfrac{\gamma}{A} = G(1-n) - \beta G'(1-n)\dfrac{F(n)}{F'(n)}$.

RHS is strictly decreasing in *n*. Hence, n is strictly *increasing* in *A*.

Message: *Another Caution for Cross-Country Study!*

- *In a closed economy* (or a sufficiently large economy), a higher A helps industrial development, because it brings down the agricultural prices and releases the labor to the M-sector. Many historians believe the Agricultural Revolution was an essential pre-condition for the British Industrial Revolution.
- *In a small open economy,* a higher A, without an offsetting decline in the agricultural prices, drives up the wage rate, and squeezes out the M-sector, deterring industrial development. Similarly, natural resource abundance could lead to a "resource curse," or "staple trap," or "Dutch Disease" in an open economy. In contrast, in a resource-poor economy, industry can develop thanks to the abundant supply of "cheap labor." (Maybe, no surprise why Belgium and Switzerland industrialized first in Continental Europe and the US Industrial Revolution began in New England, not in South.)

Welfare Implication:
In the above analysis, a higher A *cannot* hurt the economy in a small open economy model (it *could* in a large open economy; recall the Immiserising Growth). However, a higher A *could* hurt even a small open economy, when the M-sector is subject to some dynamic external economies of scale. See Part V.

Exercise:

In the above model, assume that the preferences are given by $U = \left[\beta(c_A - \gamma)^\theta + (c_M)^\theta\right]^{\frac{1}{\theta}}$. Show how the analysis needs to be modified.

Notes:

Corden (1984) developed a three-sector specific factors model, with the Booming Sector (B), the Lagging Sector (L), both of which are tradeable, and the Nontraded Sector (N). He used this model to look at the effects of the resource abundance in a small open economy and provided an excellent overview of the Dutch Disease phenomena.

Jones (1979).

Political Economy Applications (unfinished)

***Exercise*: Inspired by Antras-Caballero (2009)**

Imagine a two-sector economy with two (inherently) mobile factors, capital (K) and labor (L). The ratio of capital-labor endowments is fixed at $k = K/L$. Labor is allowed to move freely across the two sectors, but for some institutional reasons, only a fraction of capital, $\theta < 1$, can go into Sector 1; $K_1 \leq \theta K$. Assume that this is binding. Then, the model is effectively a specific factor model and its factor market equilibrium condition is given by:

$$\frac{1}{k} = \frac{L_1 + L_2}{K} = \frac{\theta}{k_1} + \frac{1-\theta}{k_2}; \; w = p_1\{f_1(k_1) - k_1 f_1'(k_1)\} = p_2\{f_2(k_2) - k_2 f_2'(k_2)\},$$

in the parameter region where $\rho_1 = p_1 f_1'(k_1) > \rho_2 = p_2 f_1'(k_2)$ holds.

Exercise 1: For a small open economy case (i.e., with p_1, p_2 fixed), show that

- If $k_1 < k_2$, $\theta\uparrow \Rightarrow k_1\uparrow, k_2\uparrow, w\uparrow, \rho_1\downarrow, \rho_2\downarrow$;
- If $k_1 > k_2$, $\theta\uparrow \Rightarrow k_1\downarrow, k_2\downarrow, w\downarrow, \rho_1\uparrow, \rho_2\uparrow$,

Intuition: As the restriction is relaxed, Sector 1 expands and sector 2 shrinks. If sector 1 is more labor intensive ($k_1 < k_2$), this would increase the relative demand for labor. With the capital-labor ratio in the economy being fixed, this causes the wage-rental ratios (and the capital-labor ratios) in both sectors to rise. The opposite happens when $k_1 > k_2$.

Corollary 1A: Suppose $f_1(k) = f_2(k) = f(k)$. Then, $p_1 f'(k_1) > p_2 f'(k_2)$ and $p_1\{f(k_1) - k_1 f'(k_1)\} = p_2\{f(k_2) - k_2 f'(k_2)\}$, which imply $k_1 < k_2$ and hence $\theta\uparrow \Rightarrow k_1\uparrow$, $k_2\uparrow$, $w\uparrow$, $\rho_1\downarrow$, $\rho_2\downarrow$.

Corollary 1B: Consider the World with two trading countries, N & S, which differ only in θ, as $1 > \theta^N > \theta^S$. Then, S exports Good 2. Furthermore, if $f_1(k) = f_2(k)$, Good 2 is capital-intensive and there is an incentive for capitals to flow from N to S.

Exercise 2: Consider an autarky case, with $f_j(k) = A_j(k)^{\alpha_j}$ and $\eta \log C_1 + (1-\eta)C_2$. Show that $\theta\uparrow \Rightarrow k_1\uparrow$, $k_2\downarrow$, and $\hat{\rho}_1 < \hat{p}_1 < \hat{w} < \hat{p}_2 < \hat{\rho}_2$. (Hint: Show first that the labor allocation is independent of θ.)

Corollary 2: Consider the World with two autarky countries, N & S, which differ only in θ, as $1 > \theta^N > \theta^S$. Then, there is an incentive for capital to flow from sector 2 in S to sector 2 in N.

Note: Corollary 1B & 2 jointly suggest the possibility that protectionism reverses the direction of flows of capital (at least those in sector 2).

Duality Approach to Factor Proportion Theory

M (Nontradeable) Factors of Production:

Endowments: M-Dimensional *Column* Vectors; $V = (V_1, V_2, \ldots, V_M)^T$
Factor Prices: M-Dimensional *Row* Vectors; $w = (w_1, w_2, \ldots, w_M)$

N (Tradeable) Commodities Produced

Outputs: N-Dimensional *Column* Vectors; $X = (x_1, x_2, \ldots, x_N)^T$
Output Prices: N-Dimensional *Row* Vectors; $p = (p_1, p_2, \ldots, p_N)$

We assume ***No-Joint Production*.**

$$\Omega = \{(x, V) \mid 0 \leq x_j \leq F^j(V_j),\ \Sigma V_j \leq V\},$$

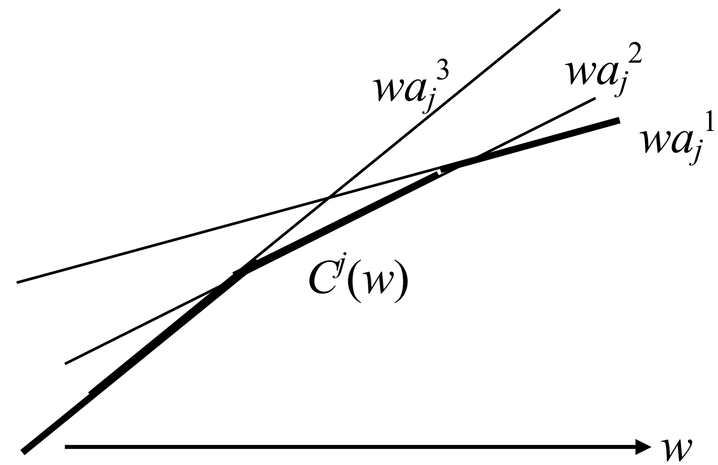
where F^j is the CRS production function of sector-j.

Exercise: Think of some examples of joint-production!

Unit Input Function of Sector-j: $a_j(w) \equiv Arg\min_{a_j}\{wa_j \mid F^j(a_j) \geq 1\}$

Unit Cost Function of Sector-j: $C^j(w) \equiv wa_j(w) = Min_{a_j}\{wa_j \mid F^j(a_j) \geq 1\}$.

Key Properties: Prove! (Similar to the expenditure/compensated demand functions.)

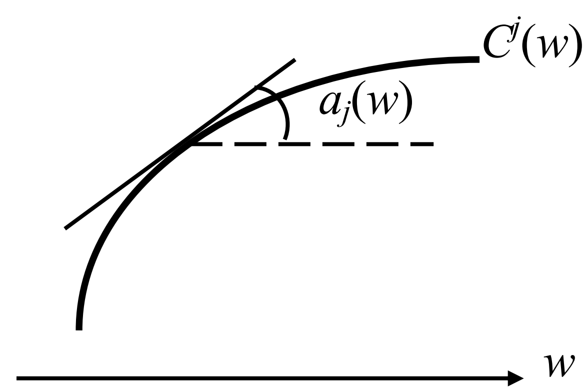


(C1) : $C^j(w) \leq wa_j$ for any a_j satisfying $F^j(a_j) \geq 1$.

(C2): $(w^1 - w^2)[a_j(w^1) - a_j(w^2)] \leq 0$.

On average, input demands are decreasing in prices.

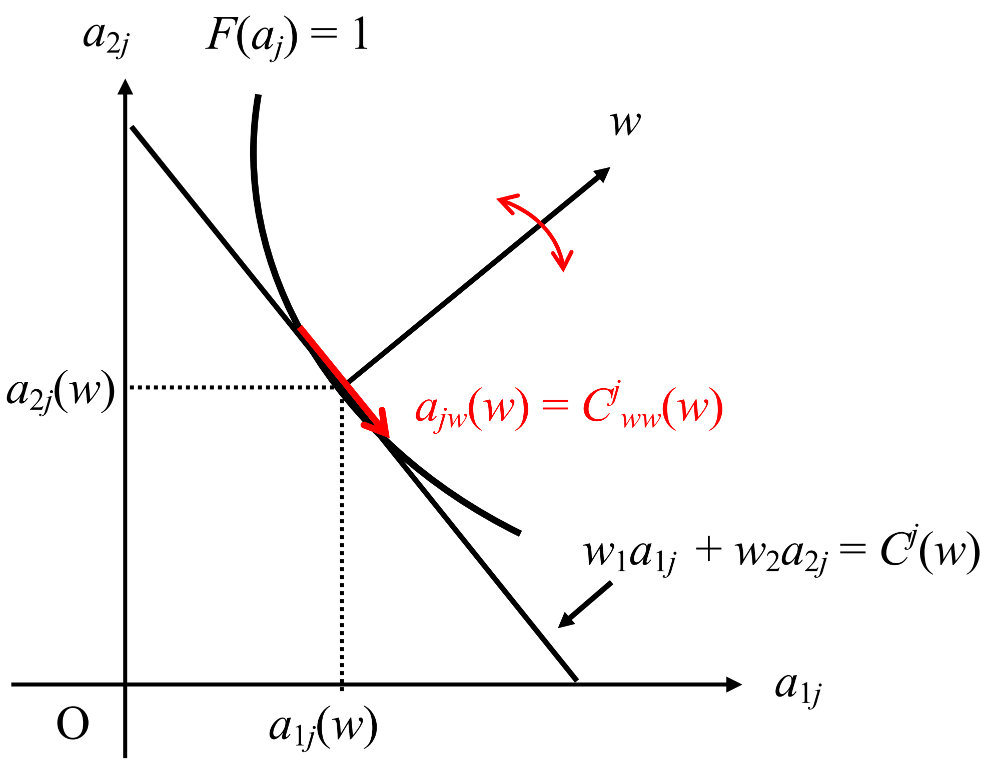
(C3) : C^j is linear homogeneous; concave in w;
$a_j(w)$ is homogeneous of degree zero.



(C4) : If C^j is differentiable in w,
$C_j(w) = wa_j(w) = wC^j_w(w)$

(C5) : If C^j is twice differentiable in w, $wC^j_{ww}(w) = 0$.

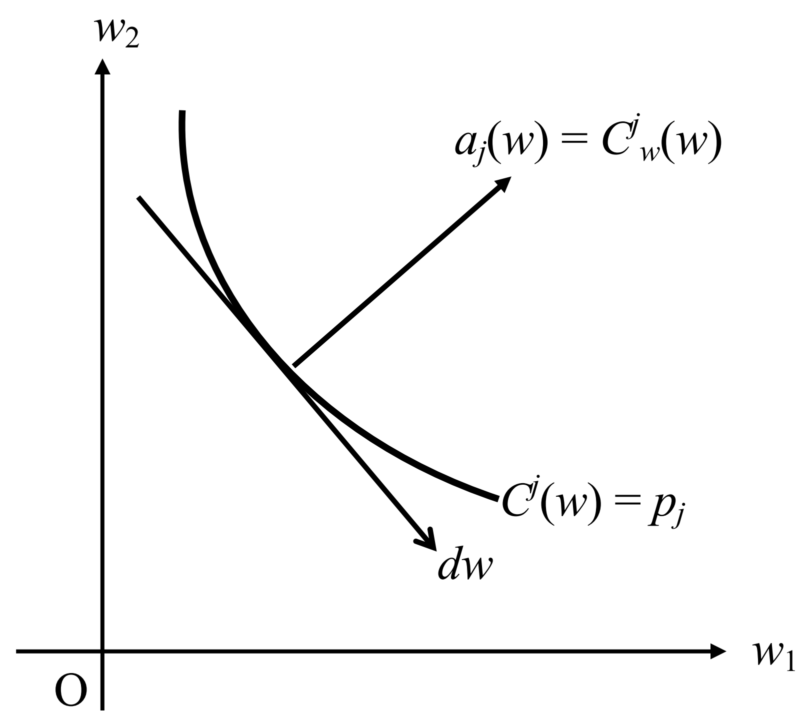
Note: $C^j_{ww}(w)$ is a *M*x*M* negative semi-definite matrix, with the rank at most equal to *M* − 1.



(C6): $\{w | C^j(w) \geq p_j\}$ is a convex set.

Note: $C^j(w)$ can also be obtained as the upper envelope of the set of linear functions, $\{wa_j | F^j(a_j) = 1\}$, and hence $\{w | C^j(w) = p_j\}$ may be viewed that the maximum combination of the factor prices that this sector can offer without making losses. For this reason, $\{w | C^j(w) = p_j\}$ is often called the *Factor Price Frontier* (FPF).

Its slope, *dw*, is orthogonal to $a_j(w)$, because $dwC^j_w(w) = 0$. The FPF is more curved with more substitutable factors. (For the Leontieff case, the FPF is a straight line.)



Factor Market Equilibrium: Given p,

(Price = Cost): $p_j \le C^j(w) = wC^j_w(w) = \sum_{i=1}^{M} w_i a_{ij}(w)$, with "=" if $X_j > 0$.

(Resource Constraint): $\sum_{j=1}^{N} a_{ij}(w) X_j \le V_i$, with "=" if $w_i > 0$.

where $a_{ij}(w)$ is the units of factor-*i* used in producing one unit of good-*j*.

In vector notation,

(P=C): $p \le wA(w)$, $X \ge 0_N$, & $[p - wA(w)]X = 0$.

(RC): $A(w)X \le V$, $w \ge 0_M$, & $w[A(w)X - V] = 0$.

where $A(w) \equiv [a_{ij}(w)] = [C^j_w(w)]$ is an MxN the unit input matrix.

Gains from (Free) Trade: An Alternative Proof

In Part 1, we saw the following proof that Free Trade is better than Autarky:

$$E(p^F, U^A) \leq p^F c^A = p^F x^A \leq p^F x^F = R(p^F,V) = E(p^F,U^F) \rightarrow U^A \leq U^F.$$

Here is the proof using the cost functions, instead of the revenue function.

Since $p^F = w^F A(w^F) \leq w^F A(w^A)$.

$$E(p^F, U^A) \leq p^F c^A = p^F x^A \leq w^F A(w^A) x^A = w^F V = E(p^F,U^F) \rightarrow U^A \leq U^F.$$

The next theorem shows that Factor Market Equilibrium can be computed as a solution to the simple convex minimization problem.

Duality Theorem: $R(p,V) = Min_w \left\{ wV \middle| C^j(w) \geq p_j \text{ for all } j \right\}$

Proof: Write the Lagrangian of the minimization problem above as follows:

$$\mathcal{L}(w) = wV + \sum_{j=1}^{N} x_j \left(p_j - C^j(w) \right)$$

with x_j being the Lagrange multiplier for the j-th constraint. Since this is a convex problem (the objective function, wV, is linear and the constraint is an intersection of convex sets, hence convex), its solution can be fully characterized by the f.o.c.

$$V_i \geq \sum_{j=1}^{N} a_{ij}(w) x_j; \qquad w_i \geq 0; \quad \sum_{j=1}^{N} x_j \left(p_j - wC_w^j(w) \right) = 0.$$

This is equivalent to the Factor Market Equilibrium by letting $x = X$ and w_i the equilibrium outputs and factor prices. Therefore,

$$wV = wA(w)X = pX = R(p,V). \qquad \text{Q.E.D.}$$

Remarks:

#1: The minimizing w is unique, unless the convex set, $\left\{ w \middle| C^j(w) \geq p_j \text{ for all j} \right\}$, has a flat boundary and V happens to be orthogonal to it.→ $R(p,V)$ is differentiable at almost all V and $w = R_V(p,V)$.

#2: Unless all factor price frontiers meet the axes at positive angles, we have $w >> 0$, which implies $A(w)X = V$. In what follows, this will be assumed to be the case.

#3: For $M \leq N$. If at least M goods are produced,

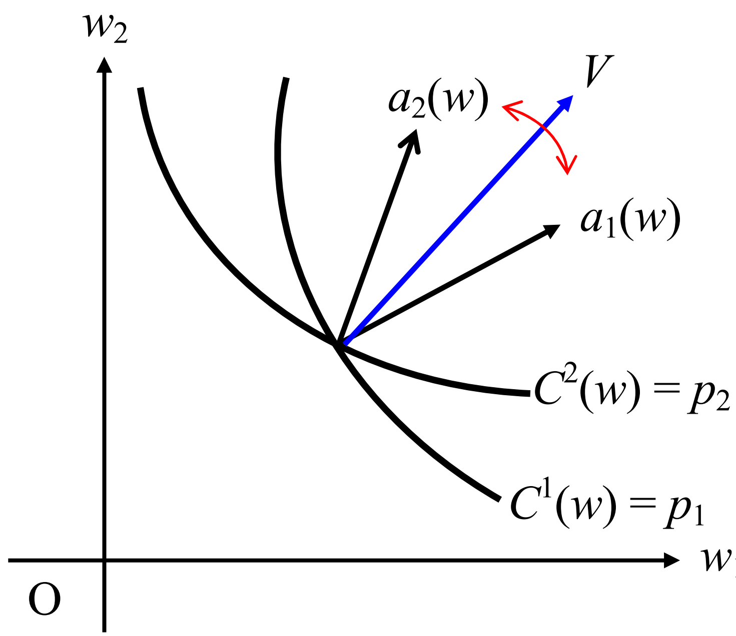
$$p_j = C^j(w) = \sum_{i=1}^{M} w_i a_{ij}(w)$$

holds for M j's and

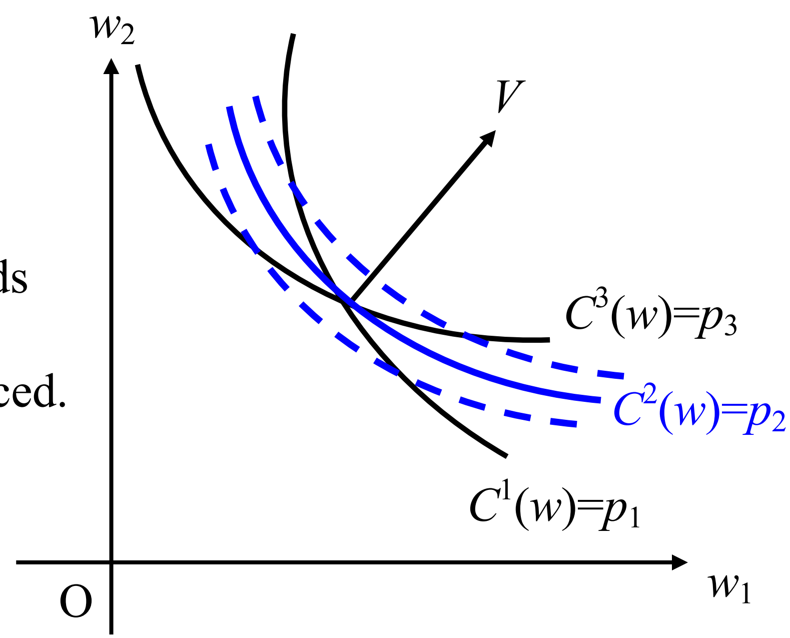
$$V_i = \sum_j a_{ij}(w) x_j .$$

Thus, V belongs to the interior of the positive cone spanned by these M vectors, $\left\{ a^j(w) \right\}$.

A change in V would not affect w.
($R_{vv} = w_v = 0$ locally).



#4: When $M < N$, $R(p,V)$ may not be differentiable w.r.t p.



In Figure, when three FPF's intersect, all three goods may be produced. Starting from this situation,

- A small rise in p_2 → Goods 1&3 will not be produced.
- A small fall in p_2 → Good 2 will not be produced.

Recall also the Ricardian model (M = 1).

#5: When $N < M$, $R(p,V)$ is differentiable w.r.t p and $x = R_p$ exists. $R_V(p,V)$ generally depends on V and hence $x_V(p,V) = R_{pV}(p,V) = [R_{Vp}(p,V)]^T = [w_p(p,V)]^T$ also depends on V.

Implication: To see the effects of factor supply changes on the output, or the effects of good prices on factor prices, one needs to solve for the entire general equilibrium.

Recall the Specific Factors Model ($M = N+1 > N$).

#6: When $N = M$, and if all the goods are produced, R_p exists and $x_V(p,V) = R_{pV}(p,V) = [R_{Vp}(p,V)]^{\mathrm{T}} = [w_p(p,V)]^{\mathrm{T}}$ does not depend on V (Note #3 above).

In fact, from $A(w)X = V$,

$$\frac{\partial x}{\partial V} = \left[\frac{\partial w}{\partial p}\right]^T = [A(w)]^{-1}.$$

Implication: To see the effects of factor supply changes on the output, or the effects of good price changes on factor prices, we do not have to solve for general equilibrium. All we need to know is the technology used, $A(w)$.

This feature makes the case of $M = N$ convenient and tractable. It also shows that how special the case of $M = N$ is. This is indeed a serious problem because we do not know how to count "goods" and "factors."

Nevertheless, let's look at the most influential (perhaps too influential) trade model, the Heckscher-Ohlin model, which assumes that $M = N = 2$.

The Heckscher-Ohlin Model (*N* = *M* = 2).

Given $p = [p_1, p_2]$,

(PC): $p_j \le C^j(w) = w_1 a_{1j}(w) + w_2 a_{2j}(w)$, with the equality when $x_j > 0$.

(RC): $$\begin{bmatrix} a_{11}(w) & a_{12}(w) \\ a_{21}(w) & a_{22}(w) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

where $a_{ij}(w)$ is the units of factor-*i* used in producing one unit of good-*j*.

Assumption (No Factor Intensity Reversal):

$a_1(w) \equiv \begin{bmatrix} a_{11}(w) \\ a_{21}(w) \end{bmatrix}$ and $a_2(w) \equiv \begin{bmatrix} a_{12}(w) \\ a_{22}(w) \end{bmatrix}$ are linearly independent at all *w* (i.e., the matrix is non-singular, or invertible, at any *w*.)

Without loss of generality, $\dfrac{a_{11}(w)}{a_{21}(w)} > \dfrac{a_{12}(w)}{a_{22}(w)}$ or $\Delta \equiv a_{11}(w)a_{22}(w) - a_{12}(w)a_{21}(w) > 0$.

Thus, good-*j* is more factor-*j* intensive than good-*i*≠*j*. (With FIR, the factor-intensity of goods depends on the factor prices.)

Factor Intensity Reversal (FIR): A Graphic Illustration

Factors are more substitutable in sector-2 than in sector-1.
With $V = V^A$, $w = w^A$, where sector-2 is more factor-1 intensive than sector-1.
With $V = V^B$, $w = w^B$, where sector-2 is more factor-2 intensive than sector-1.

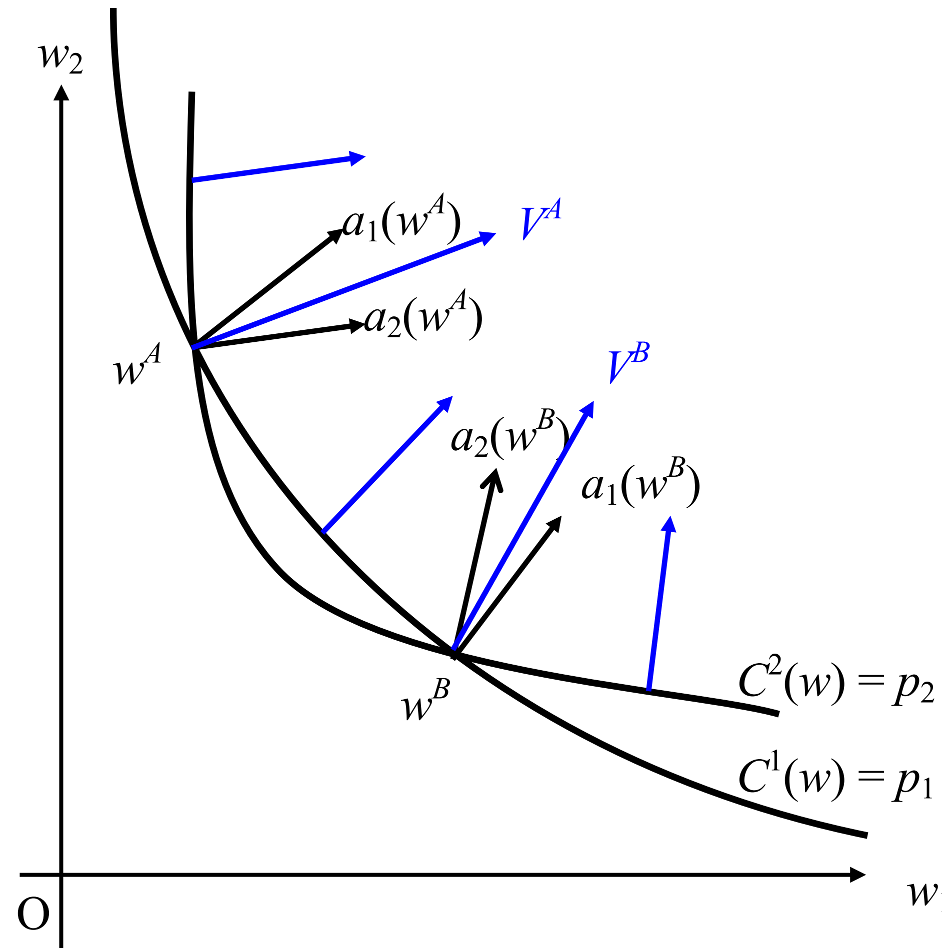
Example:

Factor-1: Capital;
Factor-2: Labor

Sector-1: Industry
Sector-2: Fishing

Country A: Capital Abundant
Country B: Labor Abundant

Fishing is more capital (labor)-intensive than industry in a high (low) wage country.



Exercises:

Show that FIR occurs whenever the two sectors both have CES technologies with different elasticities of substitution.

Show that the outputs respond *nonmonotonically* to an increase in the supply of one factor (holding the supply of the other factor).

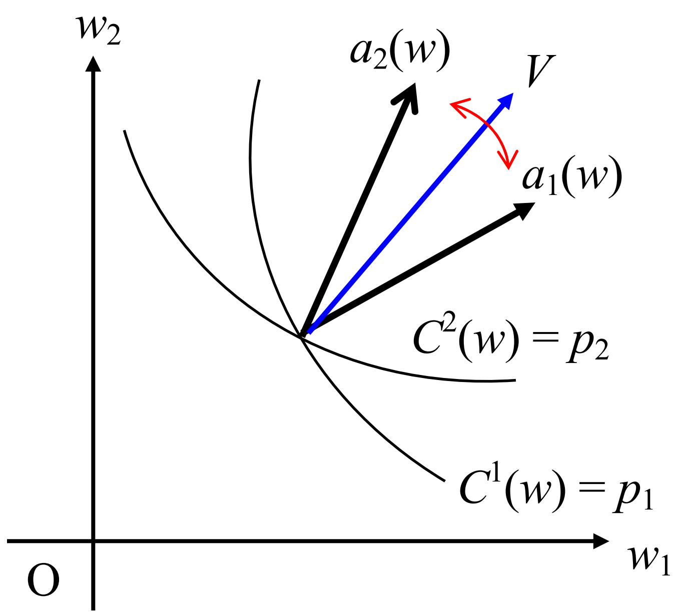
Question #1 (Diversification): When are both goods produced for a given p?

When V is in the interior of the positive cone spanned by

$$a_1(w) \equiv \begin{bmatrix} a_{11}(w) \\ a_{21}(w) \end{bmatrix} \text{ and } a_2(w) \equiv \begin{bmatrix} a_{12}(w) \\ a_{22}(w) \end{bmatrix}, \text{ where}$$

w is determined uniquely by (P=C) with the equality:

$$\begin{bmatrix} p_1 & p_2 \end{bmatrix} = \begin{bmatrix} C^1(w) & C^2(w) \end{bmatrix} = \begin{bmatrix} w_1 & w_2 \end{bmatrix} \begin{bmatrix} a_{11}(w) & a_{12}(w) \\ a_{21}(w) & a_{22}(w) \end{bmatrix}.$$



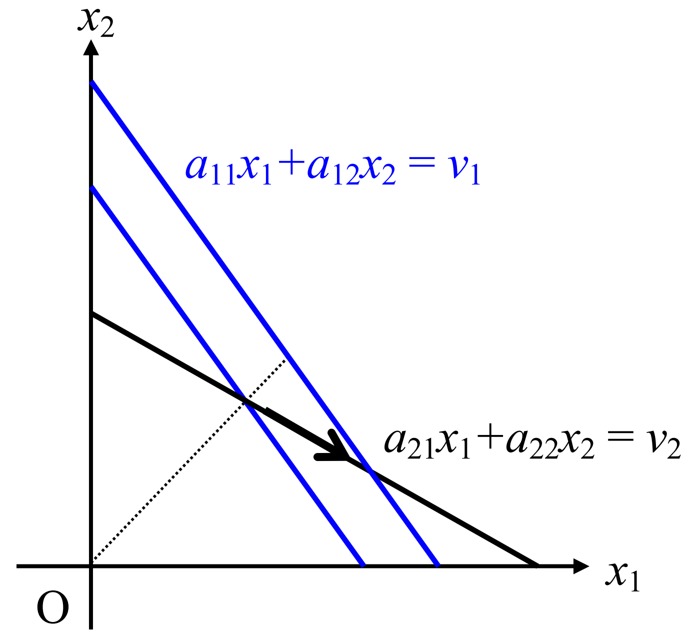
Note: This cone is often called, "the diversification cone." With FIR, there are more than one diversification cone.

Corollary: (Factor Price Insensitivity) A small change in V does not affect w (because V remains in the interior of the diversification cone).

Question #2 (Rybczynski Effect): When both goods are produced, how does a (small) change in V affect the outputs?

Since a small change in V does not change w, the input matrix is fixed. Then, from (RC),

$$\begin{bmatrix} \partial x_1 \\ \partial x_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} \begin{bmatrix} \partial v_1 \\ \partial v_2 \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} a_{22} & -a_{21} \\ -a_{12} & a_{11} \end{bmatrix} \begin{bmatrix} \partial v_1 \\ \partial v_2 \end{bmatrix}.$$



Exercise: From the above equation, prove that

$$\hat{v}_i > \hat{v}_j \Rightarrow \hat{x}_i > \hat{v}_i > \hat{v}_j > \hat{x}_j.$$

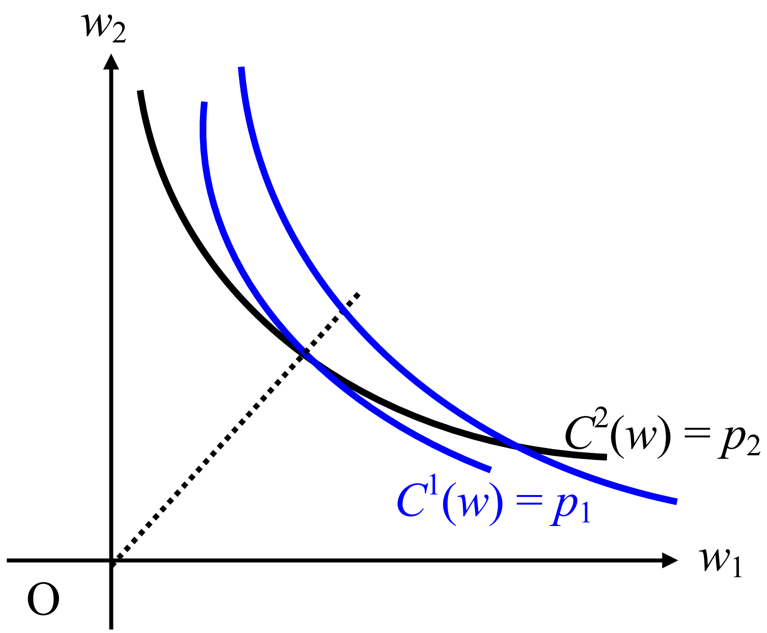
Notes:

- The above inequality is often called the *Magnification Effect*, which shows the unbalanced nature of factor increases.
- Figure illustrates the case of $\hat{v}_1 > \hat{v}_2 = 0$, which implies $\hat{x}_1 > \hat{v}_1 > \hat{v}_2 = 0 > \hat{x}_2$. Because the equilibrium moves along the resource constraint for factor-2, $a_{21}x_1 + a_{22}x_2 = v_2$, this line is often called the Rybczynski Line for factor-1.

Question #3 (Stolper-Samuelson Effect): When both goods are produced, how does a (small) change in p affect the factor prices?

From $[p_1 \quad p_2] = [C^1(w) \quad C^2(w)]$, $\partial p = \partial w A(w)$ or

$$[\partial w_1 \quad \partial w_2] = \frac{[\partial p_1 \quad \partial p_2]}{\Delta}\begin{bmatrix} a_{22}(w) & -a_{21}(w) \\ -a_{12}(w) & a_{11}(w) \end{bmatrix}.$$



Exercise: From the above equation, prove

$$\hat{p}_i > \hat{p}_j \Rightarrow \hat{w}_i > \hat{p}_i > \hat{p}_j > \hat{w}_j.$$

Notes:

- The above inequality is often called the *Magnification Effect*. It implies the owners of the factor gain (lose) unambiguously when the relative price of the good that uses this factor intensively goes up (down).
- Figure illustrates the case of $\hat{p}_1 > \hat{p}_2 = 0$, which implies $\hat{w}_1 > \hat{p}_1 > \hat{p}_2 = 0 > \hat{w}_2$.
- This figure also illustrates the effects of the Hick-neutral technical progress in sector 1.
- When sector-1 is import-competing, this figure also shows the effects of the import tariffs (in the absence of the Metzler paradox), the original question that motivated SS.

Notice the close connection between the Rybczynski and Stolper-Samuelson Theorems. This should be expected because the Reciprocity,

$$\frac{\partial x}{\partial V} = \left[\frac{\partial w}{\partial p}\right]^T = [A(w)]^{-1}.$$

The "Hat" Algebra:

Rewrite $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ to $\begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix}\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \end{bmatrix}$,

where $\lambda_{ij} \equiv \frac{a_{ij}x_j}{v_i}$ is sector-j's share in factor-i's demand with $\lambda_{11} + \lambda_{12} = \lambda_{21} + \lambda_{22} = 1$.

Rewrite $\begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$ to $\begin{bmatrix} \theta_{11} & \theta_{21} \\ \theta_{12} & \theta_{22} \end{bmatrix}\begin{bmatrix} \hat{w}_1 \\ \hat{w}_2 \end{bmatrix} = \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \end{bmatrix}$,

where $\theta_{ij} \equiv \frac{w_i a_{ij}(w)}{C^j(w)} = \frac{\partial \log C^j(w)}{\partial \log w_i}$ is factor-i's share in sector-j's cost with

$\theta_{11} + \theta_{21} = \theta_{12} + \theta_{22} = 1$.

Exercise 1: Show that

$$0 < \lambda \equiv \lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21} = \frac{x_1}{v_1}\frac{x_2}{v_2}\left(a_{11}a_{22} - a_{12}a_{21}\right) < 1,$$

$$0 < \theta \equiv \theta_{11}\theta_{22} - \theta_{12}\theta_{21} = \frac{w_1}{p_1}\frac{w_2}{p_2}\left(a_{11}a_{22} - a_{12}a_{21}\right) < 1,$$

and

$$\frac{\theta}{\lambda} = \frac{w_1 v_1}{p_1 x_1}\frac{w_2 v_2}{p_2 x_2}.$$

Exercise 2: Show that

$$\frac{\hat{x}_1 - \hat{x}_2}{\hat{v}_1 - \hat{v}_2} = \frac{1}{\lambda} > 1$$

and

$$\frac{\hat{w}_1 - \hat{w}_2}{\hat{p}_1 - \hat{p}_2} = \frac{1}{\theta} > 1.$$

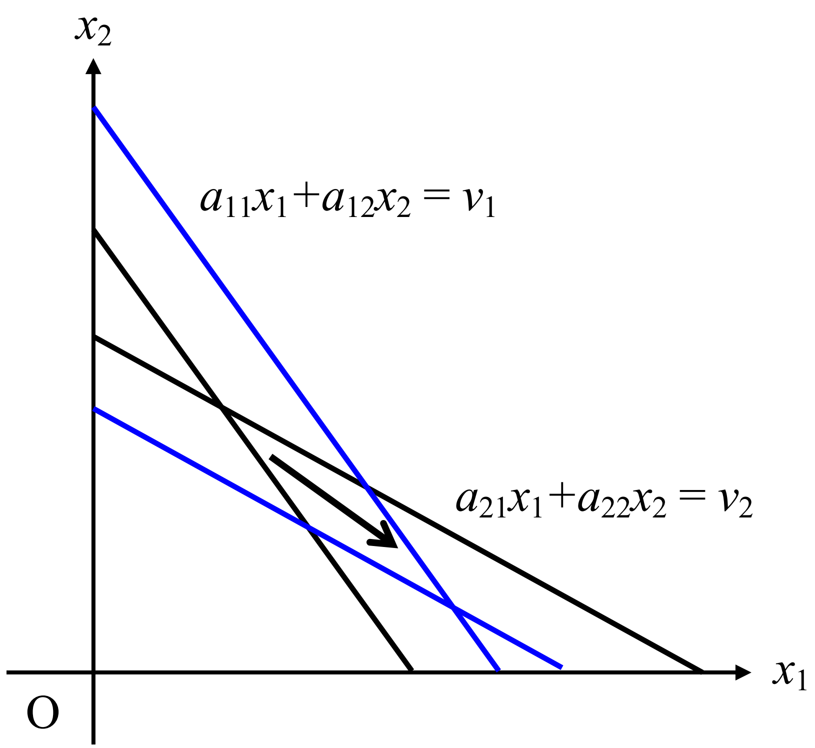
Question #4: How does a change in p affect the outputs?

$p = p_1/p_2 \uparrow \Rightarrow \hat{p}_1 > \hat{p}_2$

$\Rightarrow \hat{w}_1 > \hat{w}_2 \Rightarrow w = w_1/w_2 \uparrow$

$\Rightarrow a_{11} \downarrow, a_{12} \downarrow, a_{21} \uparrow, a_{22} \uparrow$

$\Rightarrow x_1 \uparrow, x_2 \downarrow.$



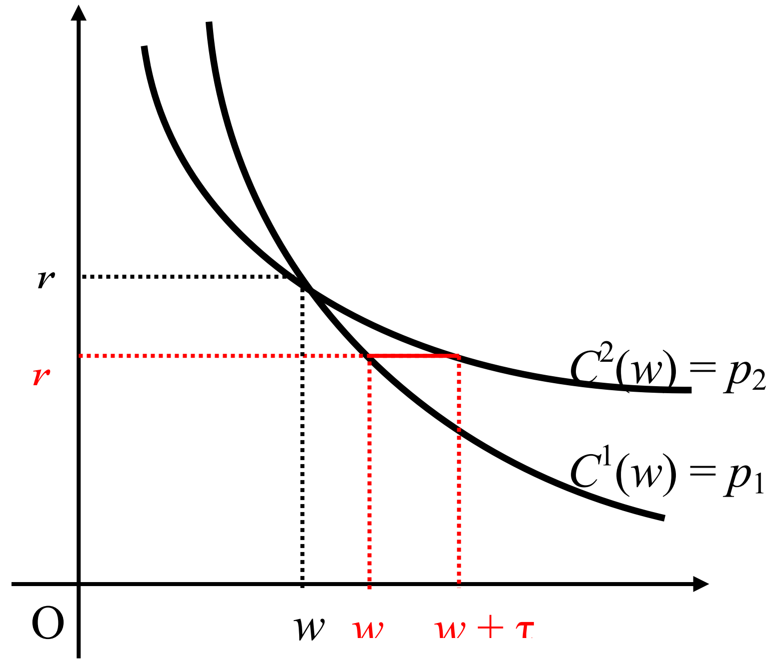
Notes:

- This should be expected from $R_{11} > 0$ and $R_{12} < 0$
- As p changes, the intersection of the two resource constraints moves along the PPF of this economy.

Factor Market Distortions in the Heckscher-Ohlin Model

Exercise: Discuss the effects of Tax (or Subsidy) on Labor (or Capital) in the Labor (or Capital)-Intensive Sectors. (Note that you have 2x2x2 = 8 combinations to discuss.)

Figure shows the effects of the labor tax in the capital intensive sector-2, which leads to a contraction of the capital-intensive sector 2 and an expansion of the labor-intensive sector 1 so much that the (after tax) wage rate will go up.



Recall the effects of the labor tax in one sector for the Specific Factor Model.

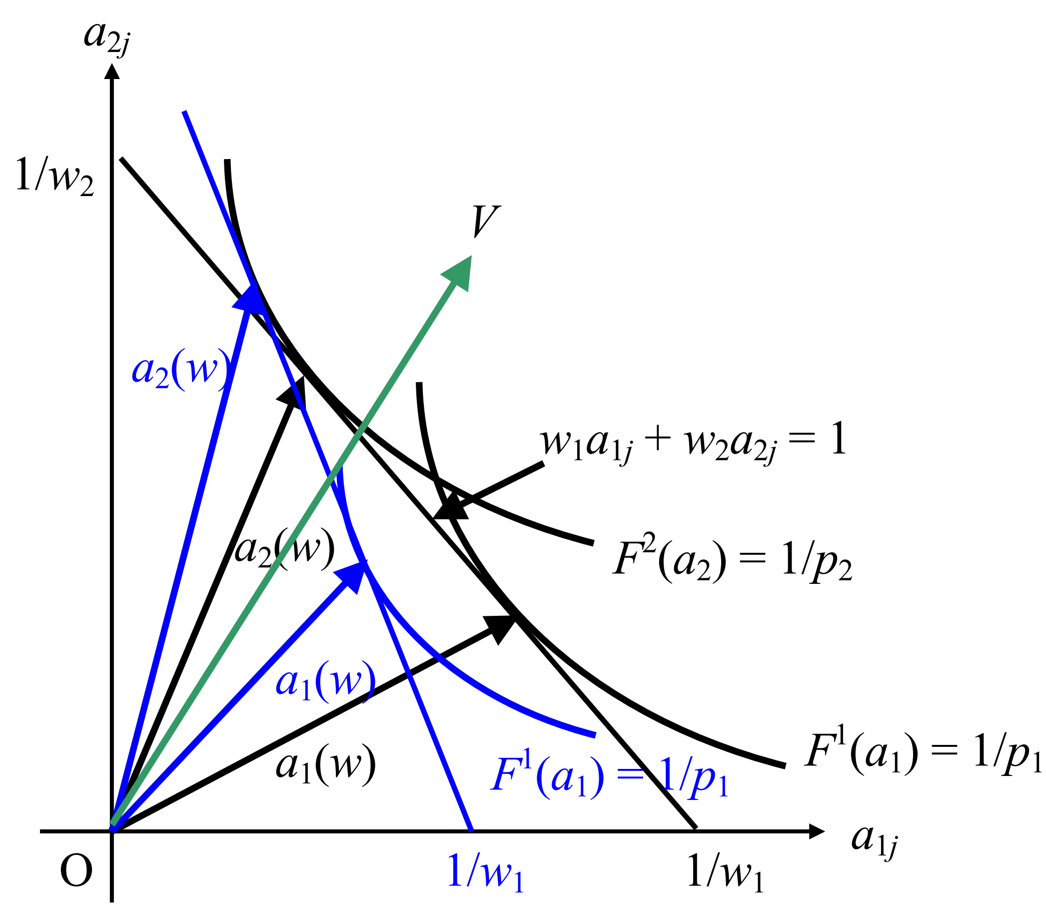
The two features of the HO model seem responsible for this (pathological result).

- All (two) factors can be reshuffled too easily across sectors.
- There are only two sectors.

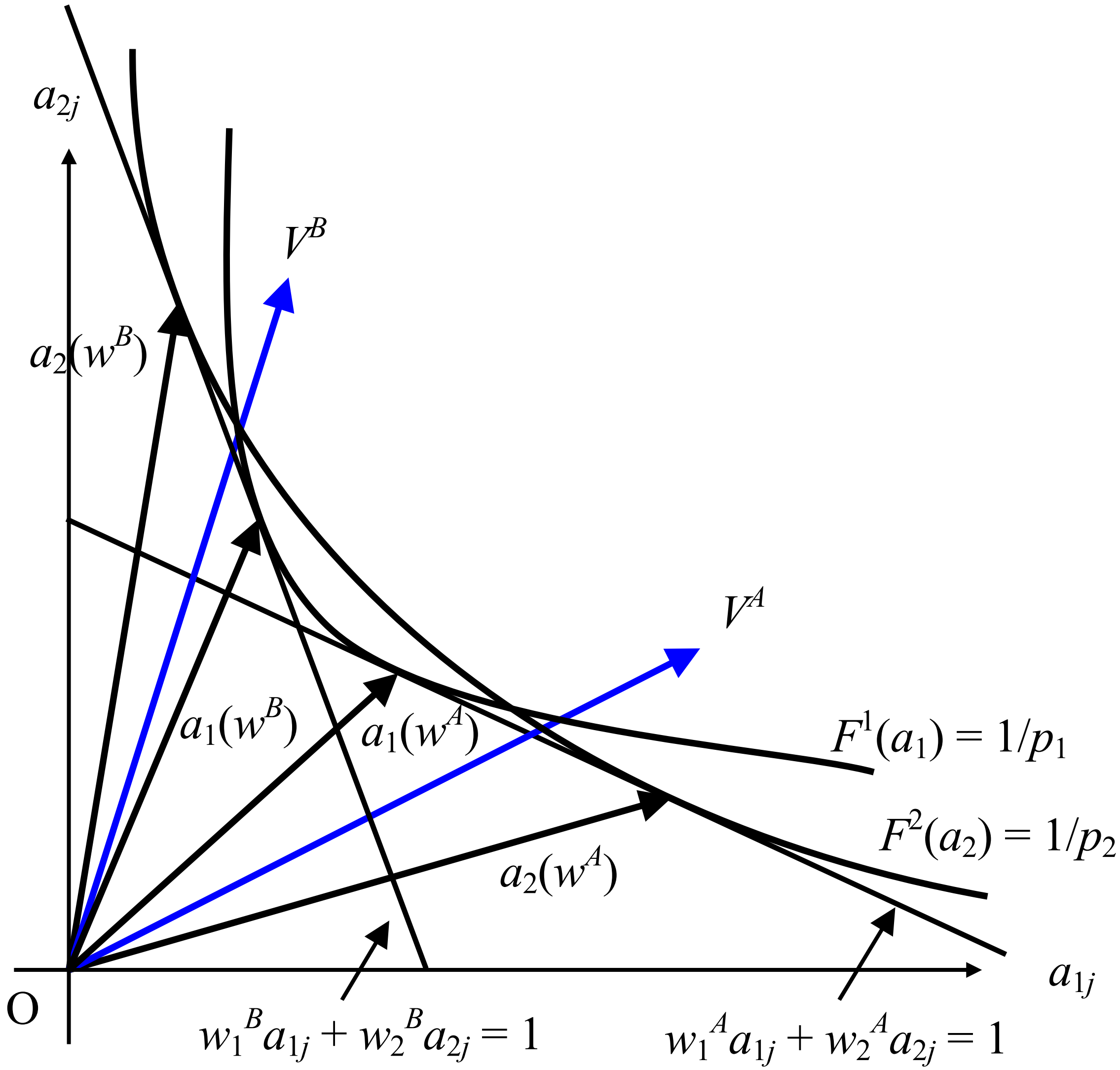
For more on factor market distortions in the HO model, see Mussa (1979).

Lerner Diagram (without FIR):

- As long as $x_1, x_2 > 0$, the unit cost line, $w_1a_{1j}+w_2a_{2j} = 1$, is tangent to the unit revenue curves of both sectors, $F^1(a_1) = 1/p_1$ and $F^2(a_2) = 1/p_2$.
- A higher p_1 (or the Hicks neutral technological progress in sector-1) moves the unit revenue curve of sector-1 closer to the origin.
- When sector-1 is more factor-1 intensive, this leads to a higher w_1 and a lower w_2, which makes both sectors use more factor-2 intensive technologies.
- That $x_1, x_2 > 0$ requires that the endowment vector, V, must belong to the diversification cone, spanned by the input vectors of the two sectors.



Lerner Diagram (with FIR): Factors are more substitutable in sector 2.



Lerner Diagram (or Primal Approach) vs. Mussa Diagram (Duality Approach)

- Many years ago, Lerner Diagram was widely used to analyze the HO model, but not since Mussa (1979), Dixit and Norman (1980) and others popularized the duality approach.
- Duality Approach and Mussa diagram (with two intersecting unit cost curves) give a more elegant exposition of the standard HO model than the Lerner diagram.
- However, I find Lerner Diagram quite useful, when dealing with the case where countries are not fully diversified or extending the HO model to many goods cases.

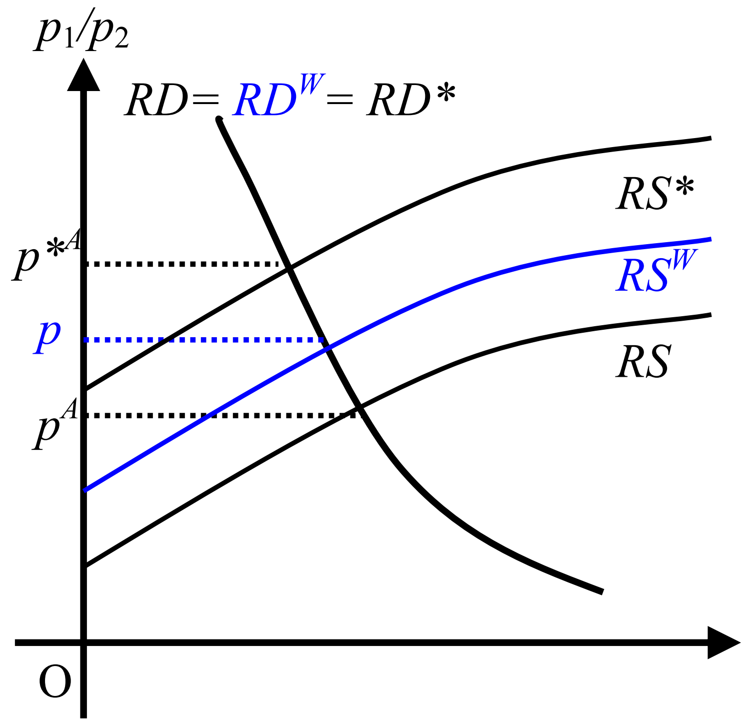
Up to this point, we have seen only some properties of the HO model for a particular country. We now turn to some properties of the HO model for the entire world economy.

For this purpose, it is assumed that the countries share the identical technologies. However, this assumption can accommodate "factor-augmenting" technological differences across countries. All we need to do is to define the factor endowments in terms of "efficiency" units. See, e.g., Trefler (1993).

Two-Country World Economy:

Consider the HO model with two countries, Home and Foreign, which differs only in the factor proportions, as $v_1/v_2 > v^*_1/v^*_2$. Then, the Rybczynski theorem implies

$$\Rightarrow \frac{x_1(p,V)}{x_2(p,V)} > \frac{x_1^*(p,V^*)}{x_2^*(p,V^*)} \text{ at all } p.$$



With the identical homothetic preferences, the two countries share the same Relative Demand Curve. Hence,

$$\Rightarrow \left(\frac{p_1}{p_2}\right)^A < \frac{p_1}{p_2} < \left(\frac{p_1^*}{p_2^*}\right)^A$$

Hence,

Patterns of Trade (Heckscher-Ohlin Theorem; Endowment Version)
Home, relatively well endowed with factor-1, exports (factor-1 intensive) Good 1 to Foreign, which is relatively well endowed with factor-2.

Note:
$\left(\frac{p_1}{p_2}\right)^A < \frac{p_1}{p_2} < \left(\frac{p_1^*}{p_2^*}\right)^A$ also implies that $\left(\frac{w_1}{w_2}\right)^A < \frac{w_1}{w_2} < \left(\frac{w_1^*}{w_2^*}\right)^A$. Hence,

Patterns of Trade (Heckscher-Ohlin Theorem: Factor Price Version)
Home, whose autarky relative price of factor-1 is lower, exports (factor-1 intensive) Good 1 to Foreign, whose autarky relative price of factor-2 is lower.

The difference between the two versions is the notion of abundance.
- The first defines the factor abundance in terms of factor proportions.
- The second defines it in terms of autarky factor prices.

The factor price version is more general as it does not require the assumption of identical preferences, but less desirable because it states in terms of the difficult-to-observe autarky prices.

Exercise:

In the Two-Country World Economy model, examine the effects of

- An increase in the Home endowment of its abundant factor; $\hat{v}_1 > \hat{v}_2 = 0$
- An increase in the Home endowment of its scare factor. $\hat{v}_1 = 0 < \hat{v}_2$.
- A proportional increase in the Home factor endowment (which keeps its factor proportion unchanged); $\hat{v}_1 = \hat{v}_2 > 0$.

on the output prices, factor prices, factor intensities, and outputs of each sector, and the welfares of the two countries. (Is Immiserizing Growth possible?) Do the answers depend on the presence/absence of FIR?

Question #5: Factor Price Equalization (FPE): Suppose that two countries that differ only in the factor proportions. In autarky, the two countries have different (relative) factor prices. Under what conditions does free trade in goods (with zero trade costs) help to equalize the factor prices?

***Related Question**:* Can Free Trade in Goods be a substitute for Free Factor Movements?

Partial equilibrium analysis with a given p: The Case of two small open economies

As long as their endowment vectors belong to the same diversification cone, FPE holds. (No FIR is not necessary for this result.) Recall that w is locally independent of V, as long as V is in the interior of a diversification cone.

Two special cases:

- Without FIR, the diversification cone is unique, so that FPE holds whenever both economies produce both goods.
- If the two factors are specific (factor-j is used only in sector-j), then FPE always holds, as the diversification cone covers the entire positive quadrant. (This is actually trivial. Assuming the existence of a tradeable good produced solely by factor-j effectively makes factor-j tradeable.)

General equilibrium analysis: The Two-country World Economy Case:

(WE)
$$p \le wA(w), \quad X \ge 0, \quad pX = wA(w)X; \qquad A(w)X = V.$$
$$p \le w^* A(w^*), \; X^* \ge 0, \; pX^* = w^* A(w^*)X^*; \qquad A(w^*)X^* = V^*$$
$$X + X^* = E_p(p, U(p, wV)) + E_p^*(p, U^*(p, w^* V^*)).$$

If all the goods are essential, (WE) holds for $w = w^* = w^I$, if and only if:

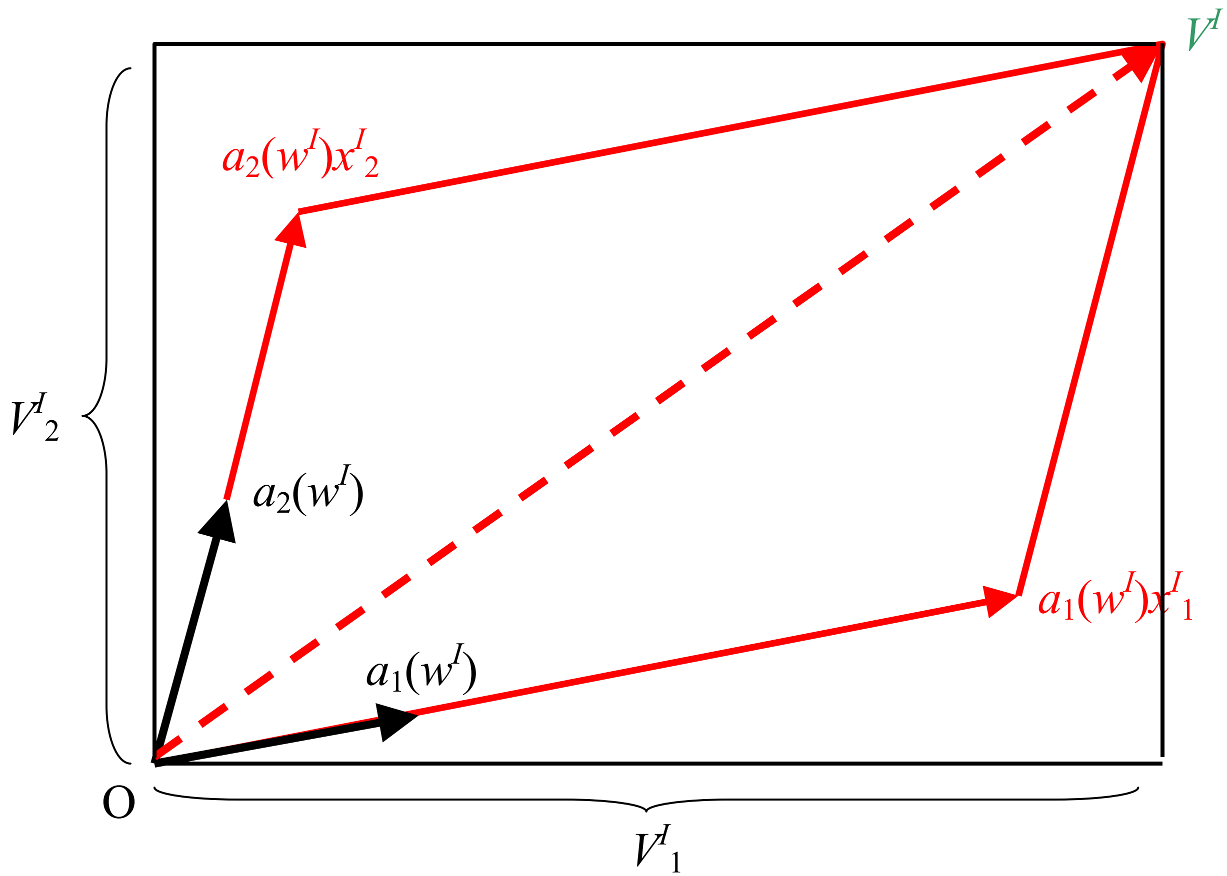
(FPE)
$$p = w^I A(w^I), \; X \ge 0, \quad A(w^I)X = V; \; X^* \ge 0, \quad A(w^I)X^* = V^*$$
$$X + X^* = E_p(w^I A(w^I), U(w^I A(w^I), w^I V)) + E_p^*(w^I A(w^I), U^*(w^I A(w^I), w^I V^*))$$

This condition, (FPE), is equivalent to the equilibrium condition of the hypothetical Integrated Economy, where all the factors are freely mobile (but the factor owners retain their preferences).

(IE)
$$p = w^I A(w^I), \qquad A(w^I)X^I = V^I \equiv V + V^*$$
$$X^I = E_p(w^I A(w^I), U(w^I A(w^I), w^I V)) + E_p^*(w^I A(w^I), U^*(w^I A(w^I), w^I V^*))$$

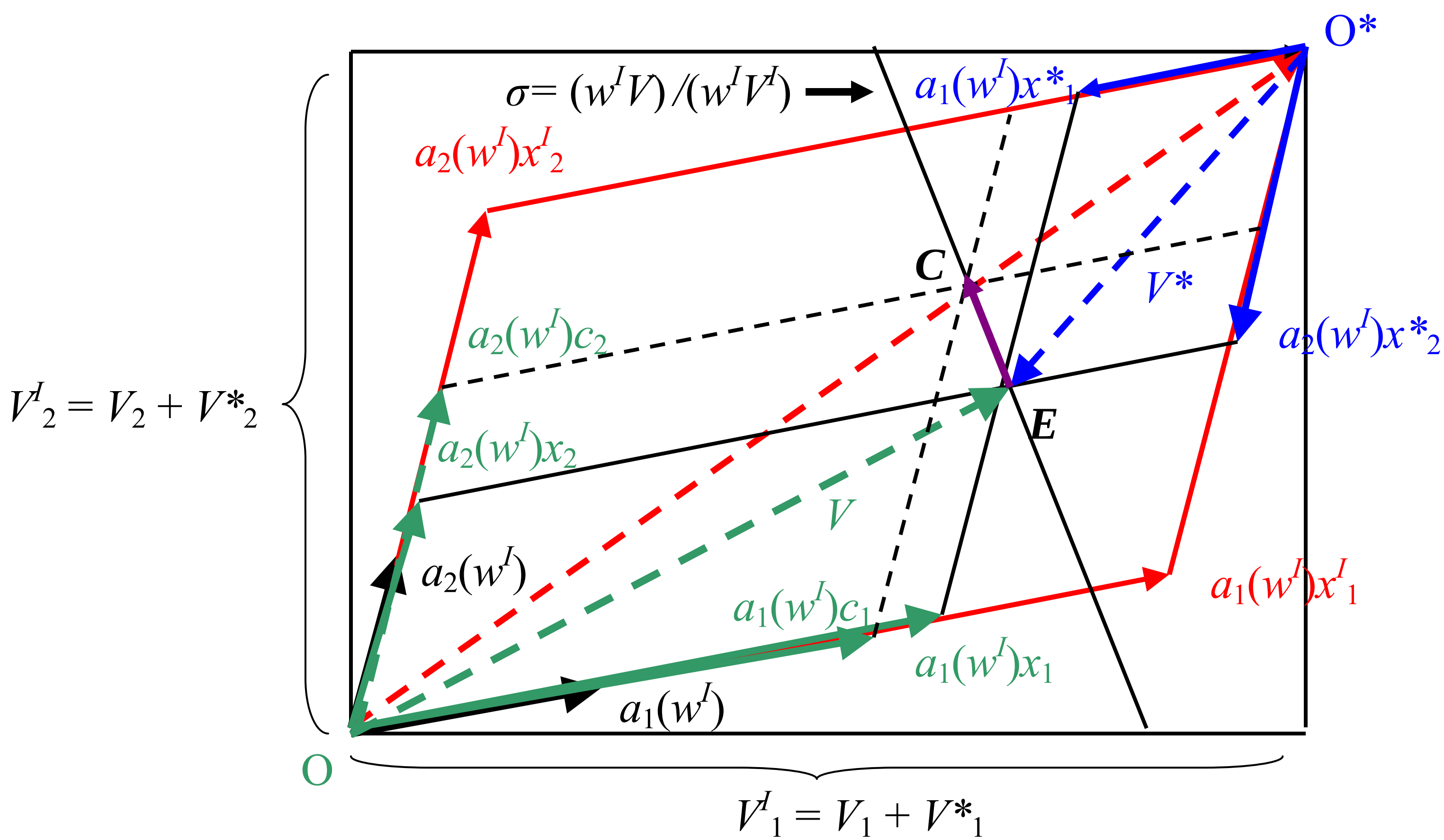
where $X^I \equiv X + X^*$, **except** that (FPE) requires $X \ge 0$ and $X^* \ge 0$ additionally.

Integrated Equilibrium: A Graphical Illustration



FPE Parallelogram (Replicating Integrated Equilibrium in a Two-Country World)**:**

When the division of the world endowment, *E,* is located inside the Red Parallelogram, Integrated Equilibrium can replicated. FPE holds. Under the identical homothetic preferences, C corresponds to consumption and the purple arrow the trade flow.



Factor Price Equalization (FPE) Theorem:

Let w^I and X^I be the equilibrium vectors of the factor prices and the outputs of the Integrated World Economy. Then, FPE holds if and only if

$$V \in \left\{ A(w^I)X \,\middle|\, 0 \le X \le X^I \right\} \text{ or equivalently } V^* \in \left\{ A(w^I)X^* \,\middle|\, 0 \le X^* \le X^I \right\}.$$

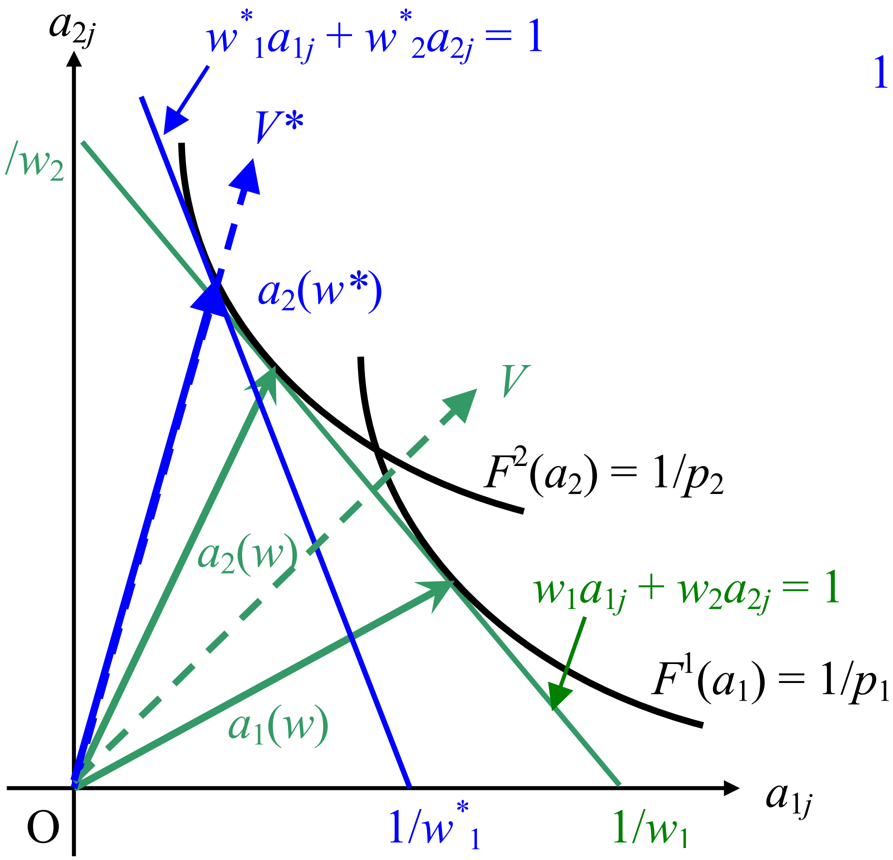
Notes:

- This theorem does not require the absence of FIR. It merely requires that the factor proportions of the two countries are not too dissimilar, so that their endowment vectors belong to the same diversification cone, spanned by the two input vectors calculated at the (hypothetical) integrated equilibrium.
- FIR might matter when the factor proportions are too dissimilar (and hence E falls outside the FPR Parallelogram).

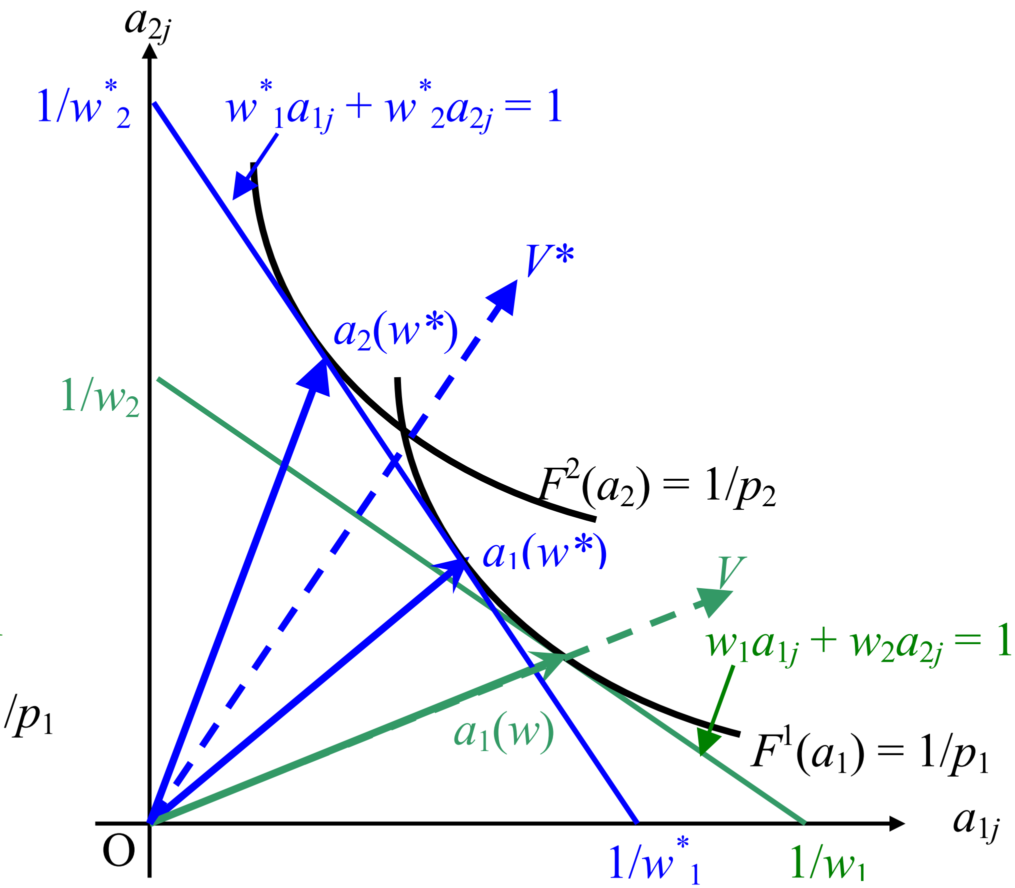
So, what happens outside the FPE Parallelogram?

From the concavity of $R(p,V)$ in V, $\dfrac{v_1}{v_2} > \dfrac{v_1^*}{v_2^*} \Rightarrow \dfrac{w_1}{w_2} = \dfrac{R_{v_1}(p,V)}{R_{v_2}(p,V)} < \dfrac{w_1^*}{w_2^*} = \dfrac{R_{v_1}(p,V^*)}{R_{v_2}(p,V^*)}$.

Without FIR: Two generic cases; both suggest that Home exports (imports) factor-1 (2) intensive Good 1 (2) to Foreign. HO Theorem holds.



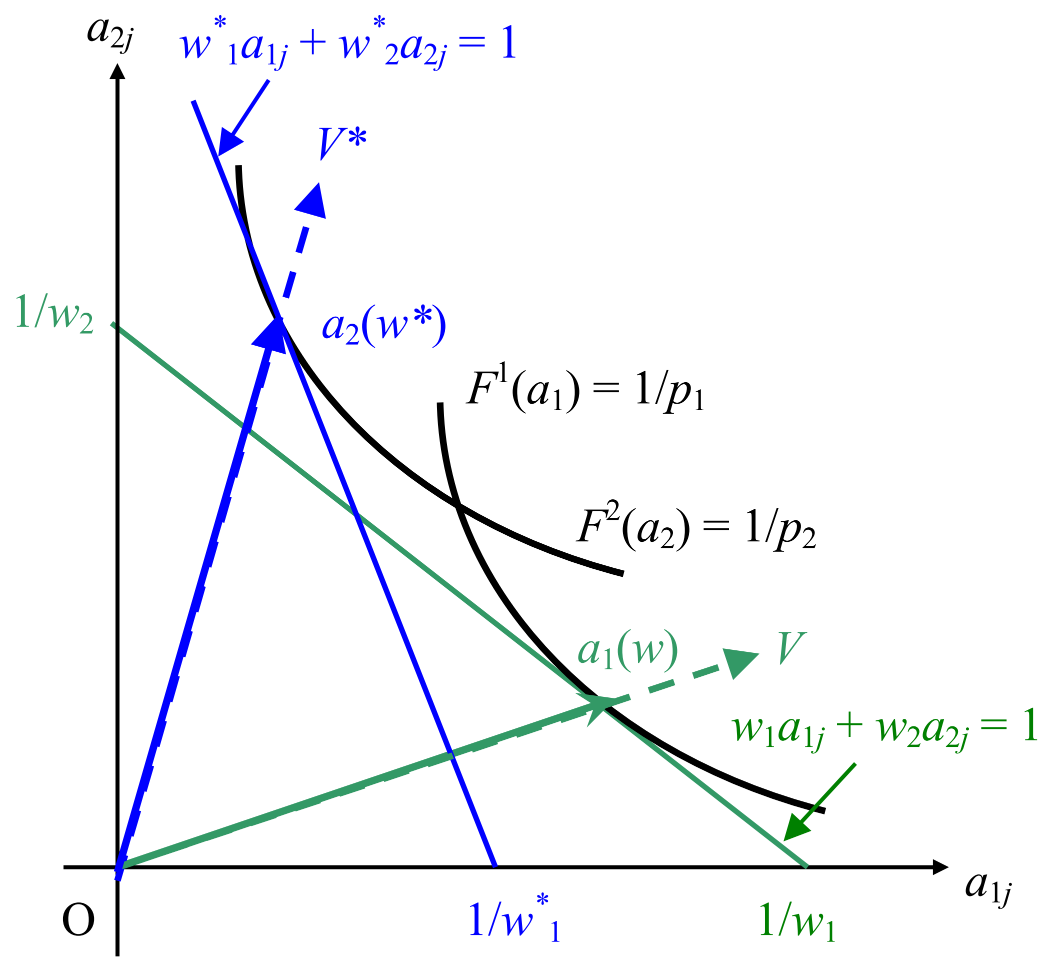
Home produces both goods.
Foreign specializes in Good 2.



Foreign produces both goods.
Home specializes in Good 1.

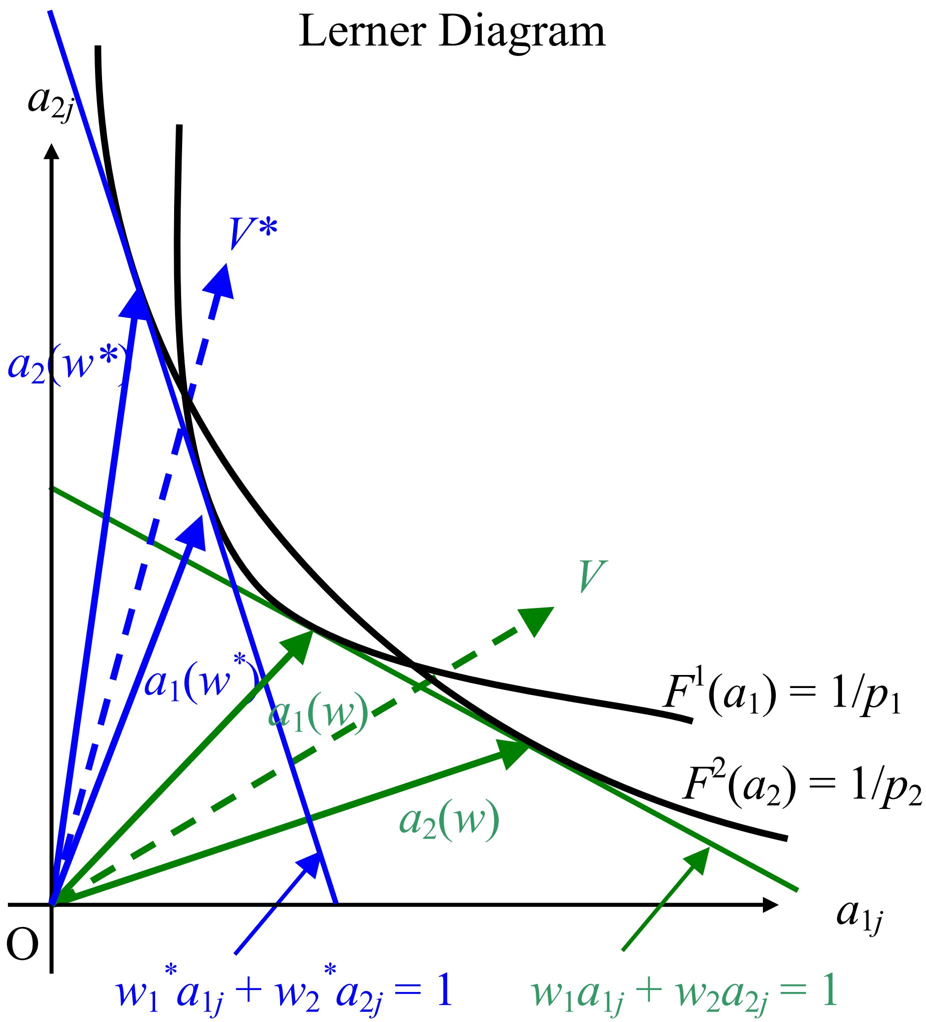
Exercise:

Explain why it is *very unlikely* that both countries specialize as shown in this Figure.

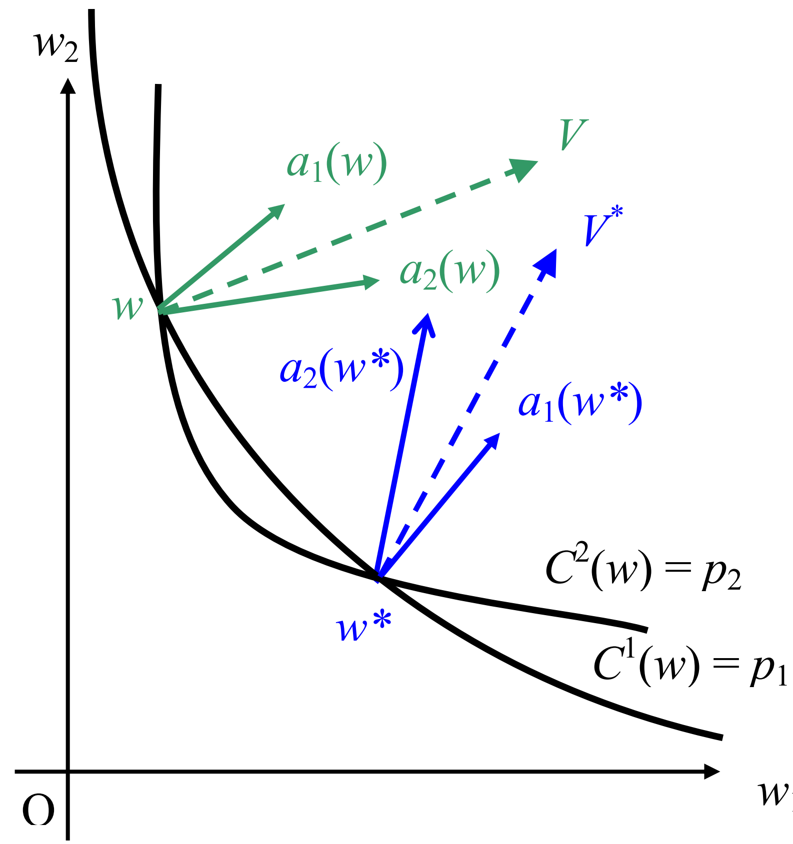


What happens outside the FPE Parallelogram (with FIR)?

Lerner Diagram



Mussa Diagram



Thus, ***without FPE <u>and</u> with FIR,***

- Both countries produce both goods.
- Depending on the volume of productions (which are determined by the exact location of each country's endowment vector within its own diversification cone), Home may export good 1 or good 2. Hence, the factor endowment differences alone cannot predict the direction of trade flows.
- Indeed, the factor-intensity of the good cannot be defined unambiguously. (At Home, good-2 is more factor-1 intensive. At Foreign, good-1 is more factor-1 intensive.)

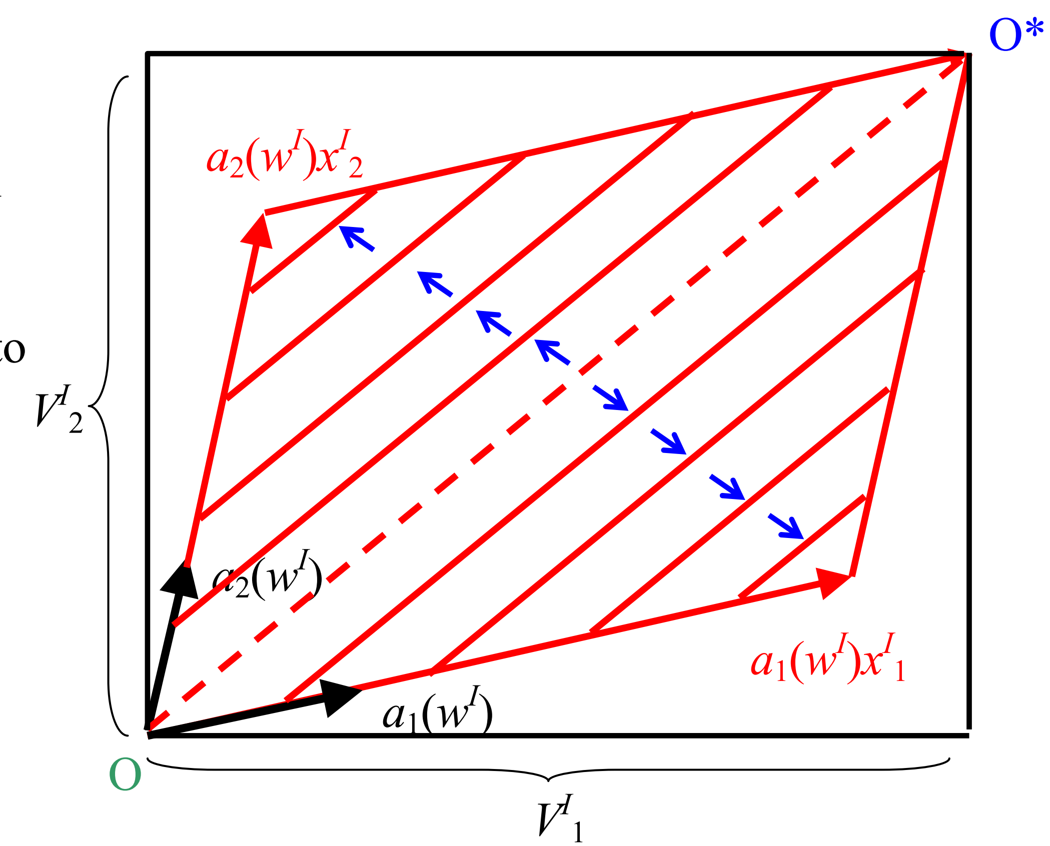
However,

- Home produces *all* goods using more factor-1 intensive technologies than Foreign.
- Regardless of the direction of trade flows, the Home export good "contains" more factor-1 relative to factor-2 than the Foreign export good.
- This suggests that the "Factor Content" of Trade is a more robust prediction of this model.

Volume of Trade:

One can show (see, e.g., Helpman-Krugman 1985, Ch.8) that, inside the FPE Parallelogram, that the iso-trade volume loci are given by lines parallel to the diagonal, and that the volume increases farther away from the diagonal. It would be more involved to derive such loci outside the Parallelogram.

I will not dwell on this, because this prediction of the HO model is not particularly robust.



High-Dimensional Factor-Proportion Theory
How much can we extend these results to general M and N?

Rybczynski Effect: How does a (small) change in V and V^* affect X and X^*?

1) For $N < M$, the R-effect is substantially weakened. (e.g., we can say very little about the effect of an increase in the mobile factor in the Specific Factors Model.)

2) For $N \geq M$, assume that at least M of N goods are produced. Then, with a small change in V from V^0 to V^1, $A(w)X^0 = V^0$, $A(w)X^1 = V^1$ with the same w(Factor Price Insensitivity).
a) If $N = M$ and $A(w)$ is invertible, $X^1 - X^0 = [A(w)]^{-1}(V^1 - V^0)$. Or, with the "hat" algebra,

$$\hat{V} = \Lambda\hat{X} \text{ or } \hat{X} = \Lambda^{-1}\hat{V}, \text{ where } \hat{V} \equiv \left\{\frac{\partial v_i}{v_i}\right\}, \ \hat{X} \equiv \left\{\frac{\partial x_j}{x_j}\right\}, \text{ and } \Lambda \equiv \{\lambda_{ij}\} = \left\{\frac{a_{ij}x_j}{v_i}\right\}.$$

We can obtain many strong results because Λ is a Markov-matrix.
b) If $N > M$, X^0 and X^1 are generally indeterminate, so that we cannot look at the R-effect for each good. The most we can say is the following co-variation:

$$(V^1 - V^0)^T A(w)(X^1 - X^0) = (V^1 - V^0)^T (V^1 - V^0) > 0.$$

Stolper-Samuelson Effect: How does a (small) change in p affect w?

For any good produced, $p = wA(w)$. If a change in p from p^0 to p^1 does not affect the set of goods produced, $p^0 = w^0A(w^0)$ and $p^1 = w^1A(w^1)$. By applying the mean value theorem to $f(w) \equiv wA(w)(p^1 - p^0)^{\mathrm{T}}$, $f(w^1) - f(w^0) = (w^1 - w^0)A(\xi)(p^1 - p^0)^{\mathrm{T}}$. Hence,

$$(w^1 - w^0)A(\xi)(p^1 - p^0)^T = (p^1 - p^0)(p^1 - p^0)^T > 0.$$

If $N = M$ and all goods are produced, $p = C(w)$, so $dp = dwA(w)$. If $A(w)$ is invertible,

$$\left[\frac{\partial w}{\partial p}\right]^T = [A(w)]^{-1} = \frac{\partial x}{\partial V}.$$

Or, using the "hat" algebra,

$$\hat{p} = \hat{w}\Theta \text{ or } \hat{w} = \hat{p}\Theta^{-1}, \text{ where } \hat{p} \equiv \left\{\frac{\partial p_j}{p_j}\right\}, \hat{w} \equiv \left\{\frac{\partial w_i}{w_i}\right\}, \text{ and } \Theta \equiv \{\theta_{ij}\} = \left\{\frac{w_i a_{ij}}{p_j}\right\}.$$

Note that Θ is a Markov-matrix.

For $M > N \geq 2$, consider the two goods produced, 1 and 2. Then, $p_1 = \sum_{i=1}^{M} w_i a_{i1}$ and $p_2 = \sum_{i=1}^{M} w_i a_{i2}$. Thus, $\hat{p}_1 = \sum_{i=1}^{M} \hat{w}_i \theta_{i1}$ and $\hat{p}_2 = \sum_{i=1}^{M} \hat{w}_i \theta_{i2}$. This means that, if $\hat{p}_1 > \hat{p}_2$, there exists two factors such that

$$\hat{w}_{i1} > \hat{p}_1 > \hat{p}_2 > \hat{w}_{i2}.$$

Recall the Specific Factors Model.

Note: The assumption of No-Joint Production is important, because it implies that any output price is the weighted average of the input prices. Thus, the output price cannot change by 10%, unless some input prices change more than 10%.

See Ethier (1983) for more on higher-dimensional versions of the Rybczynski and Stolper-Samuelson theorems.

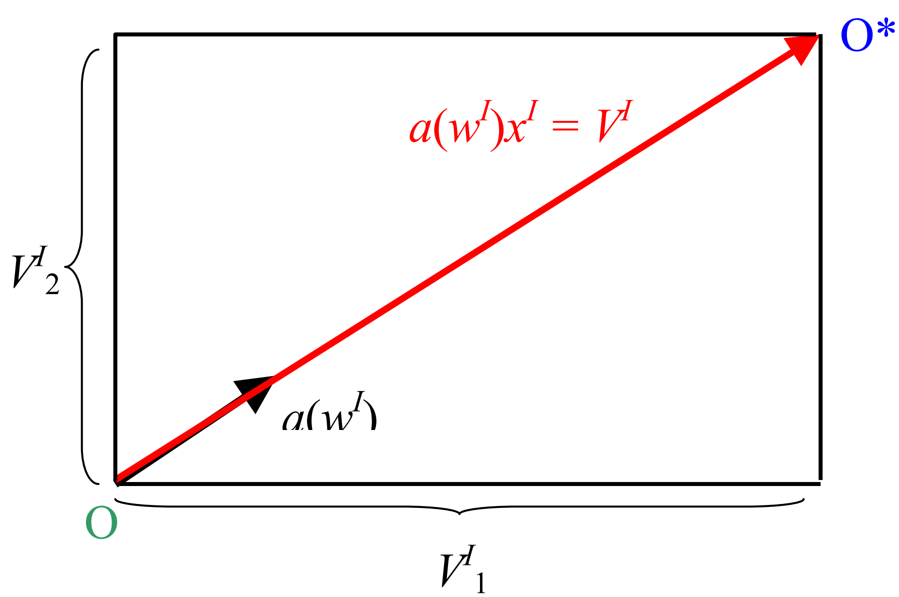
Factor Price Equalization: The FPE theorem itself holds for any dimension. That is, FPE holds iff $V \in \left\{ A(w^I)X \middle| 0 \le X \le X^I \right\}$.

For $N < M$, $A(w^I)X$ is an N-dimensional cone in an M-dimensional space. Hence, this condition is unlikely to hold.

Example 1: One-sector Neoclassical Model with Labor & Capital ($M = 2 > N = 1$).

The cross-country difference in the capital/labor ratio leads to the cross-country differences in the rate of return to capital, which can be substantial (unless without technological differences) →The Lucas (1990) Paradox

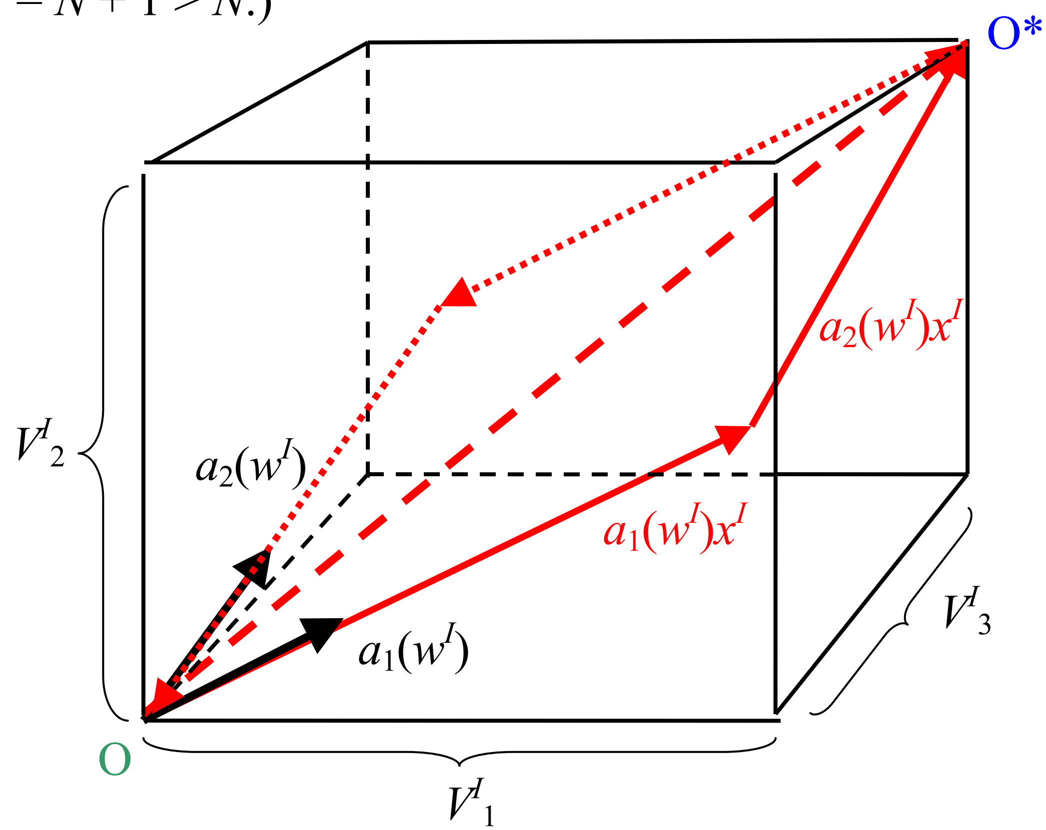
FPE Set for M = 2 > N = 1:



Example 2: Specific Factor Models ($M = N + 1 > N$.)

FPE Set for M = 3 > N = 2:

This figure is drawn under the assumption that each sector uses all three factors.



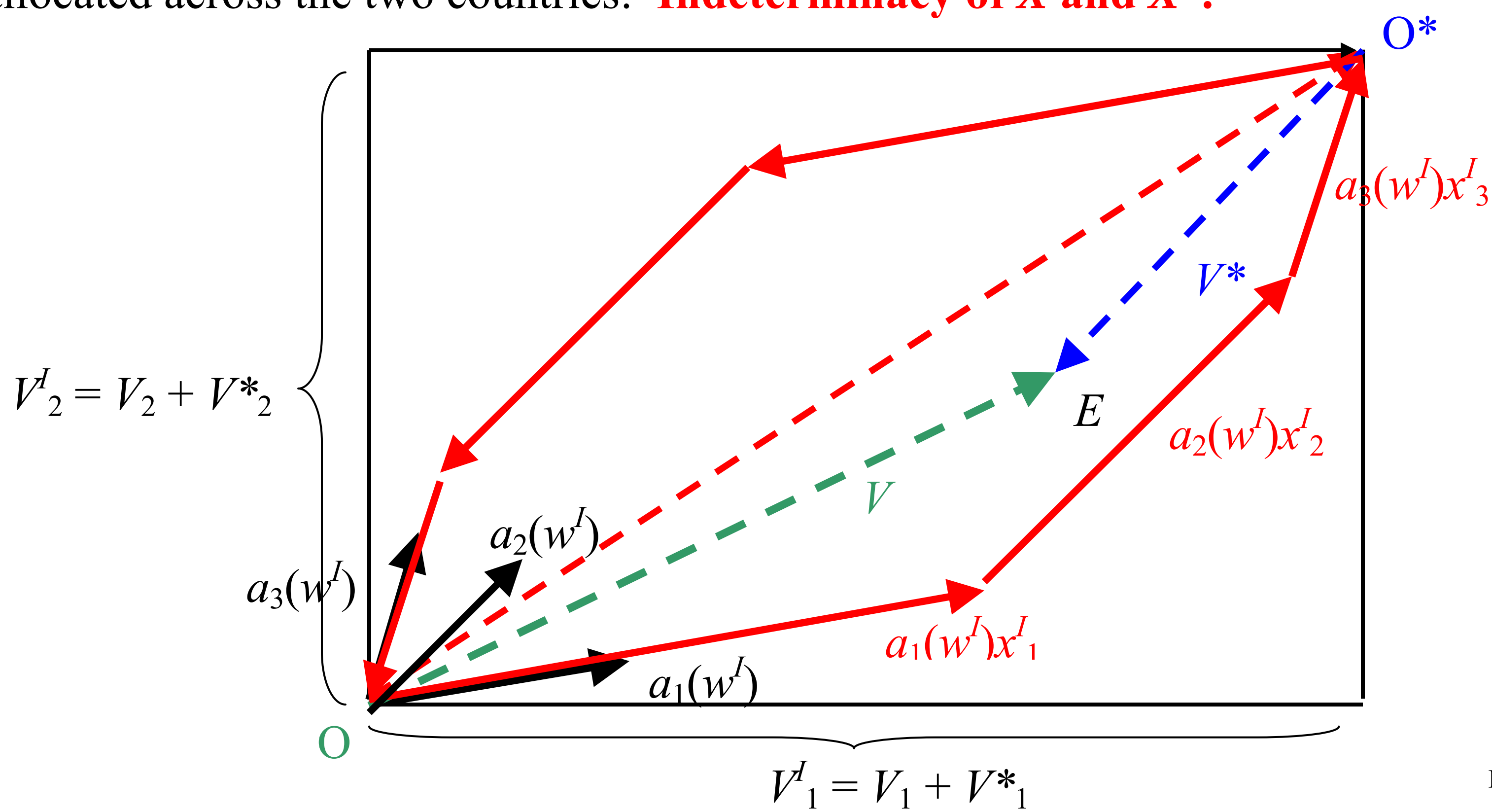
One might ask:

Even though trade may fail to equalize the factor prices, it may help the factor prices to converge. (Indeed, that was the original question asked by Ohlin. It was Samuelson who rephrased the question). The answer is not necessarily. See Dixit and Norman (1980, pp.102-105) for an example in which trade causes the factor prices to diverge.

For $N \geq M$, $A(w^I)$ can contain at least M linearly independent column vectors. If so, $A(w^I)X$ is an M-dimension. Thus, we can hope for FPE, at least when the factor proportions of V and V^* are not too dissimilar.

FPE Set for M = 2 < N = 3:

- If the division of the world endowment, E, is located inside the Red Hexagone, the integrated equilibrium can be replicated in the two-country world.
- However, with $N > M$, there are many ways in which the production activities can be allocated across the two countries: **Indeterminacy of X and X^*.**



Notes:

- The Lucas (1990) paradox may disappear in a Multi-Sector Model with Labor and Capital ($M = 2 \leq N$).

- More generally, we can hope for FPE, when the # of goods tradeable at zero cost, N_T, is no less than the # of nontradeable factors, M_N . See Ethier-Svensson (1988).

Example 1**:** Mundell (1957) showed that factor movements and trade in goods are *substitutes* in the 2x2 HO model with capital and labor, when two countries differ only in the capital/labor ratios.

➢Without Trade in Goods, but with free capital mobility, capital moves from the capital abundant country to the labor abundant country, until the factor prices and factor ratios are equalized. The result: No need to trade goods.

➢Without Capital mobility, but with Free Trade in Goods, FPE can occur, in which case no need for capital mobility.

Example 2: Komiya (1967) showed FPE in a model with 2-Factors and 3-Goods, one of which is nontradeable. See also Helpman-Krugman (1985; Ch.1.4).

A Critical Note on the "even"-case, M = N.

- One may be tempted to argue that FPE holds for $N = M$, when both countries produce all N goods and there is no FIR, i.e., $A(w)$ is non-singular or invertible at any w. Such a statement is true for $N = M = 2$, not necessarily true for $N = M > 2$.

- The fact that both countries produce all goods ensure $wA(w) = p = w^*A(w^*)$. However, this does not necessarily mean $w = w^*$ unless $f(w) \equiv wA(w)$ is a one-to-one mapping.

- For $N > 2$, the non-singularity of A(w) is not sufficient for f(w) to be one-to-one.

- A stronger condition, known as the Gale-Nikaido (1965) condition ensures that $f(w)$ is one-to-one for $N > 2$, but its economic interpretation is unclear.

- In any case, Dixit and Norman (1980) criticized this line of reasoning, because it takes p, rather than V and V^*, exogenous in spite that FPE is about the general equilibrium property of the world economy.

- I also find it uninteresting, as it relies so much on the condition, $M = N$.

Heckscher-Ohlin Theorem: How does the factor endowment difference affect the patterns of trade?

For $N \geq M$, assume *FPE,* so that $w = w^*$, $A(w)X = V$, $A(w)X^I = V^I$ and $p = wA(w)$. With the identical homothetic preferences, the Home's demand share in the world output is equal to the Home's share in the world income, $\sigma \equiv (wV)/(wV^I)$. Thus, $C = \sigma X^I$ and the Home's import vector is given by $M = C - X = \sigma X^I - X$. From $A(w)X = V$, $A(w)X^I = V^I$, the ***Factor Content of Net Trade*** is given by

Heckscher-Ohlin-Vanek Theorem: $A(w)M = \sigma V^I - V$.

If $N = M$ and $A(w)$ is nonsingular, this can be rewritten as:

Heckscher-Ohlin Theorem: $M = [A(w)]^{-1}(\sigma V^I - V)$.

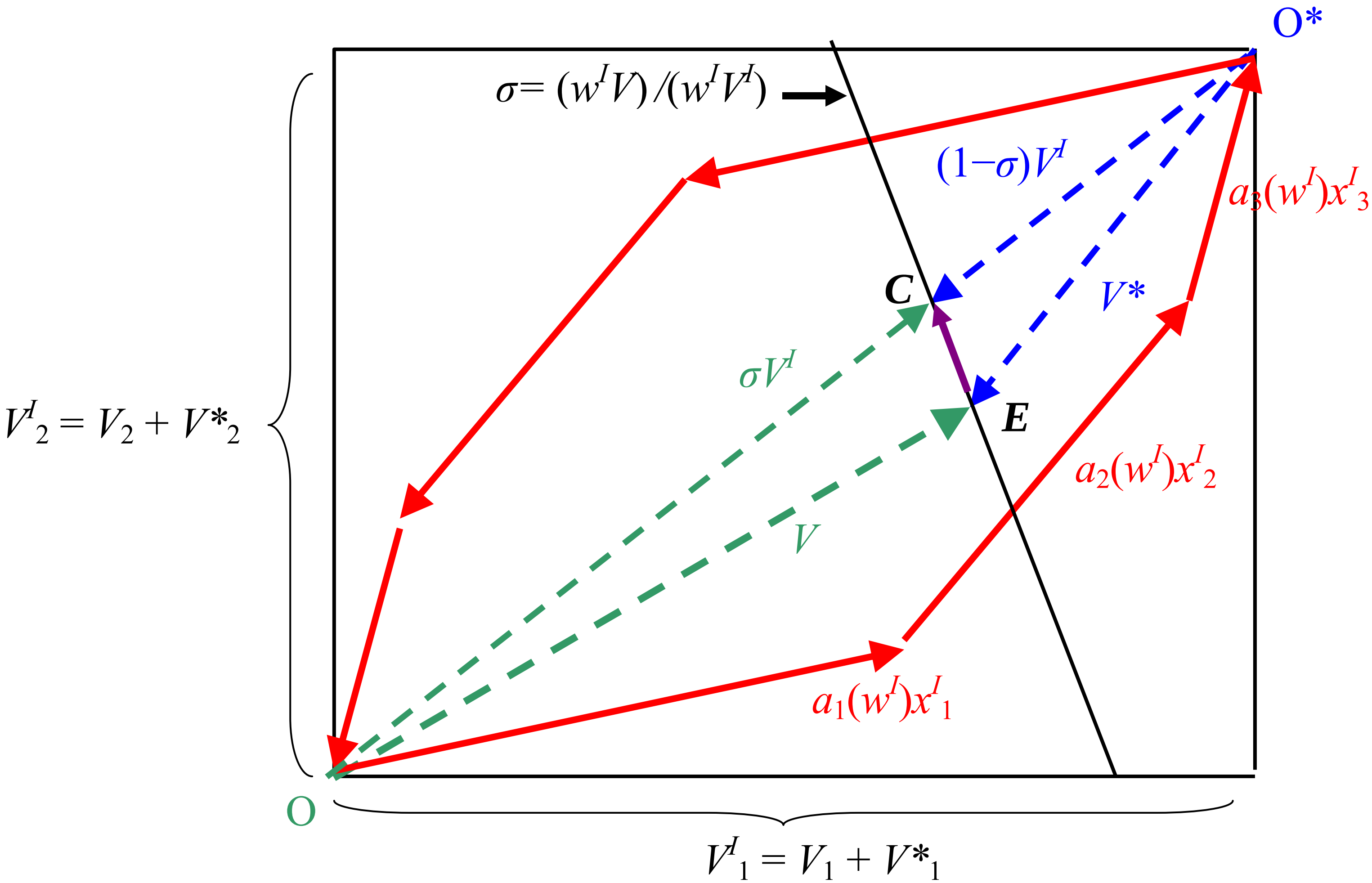
If $N > M$, X (and X^*) is indeterminate, there is no hope for predicting the patterns of trade goods-by-goods. In terms of correlation,

$$(\sigma V^I - V)^T A(w)M = (\sigma V^I - V)^T (\sigma V^I - V) \geq 0.$$

If $N < M$, HO is substantially weakened. Recall the Specific Factor Model, which has no definite prediction when the two countries differ in the endowment of the mobile factor.

Heckscher-Ohlin-Vanek Theorem: A Graphic Illustration

Purple Arrow represents the Home's Factor Content of Trade, $A(w)M = \sigma V^I - V$, which is uniquely determined. However, M is indeterminate, because X is indeterminate



Failure of the HO Theorem with *N* > *M* and FPE: A Lerner Diagram Illustration

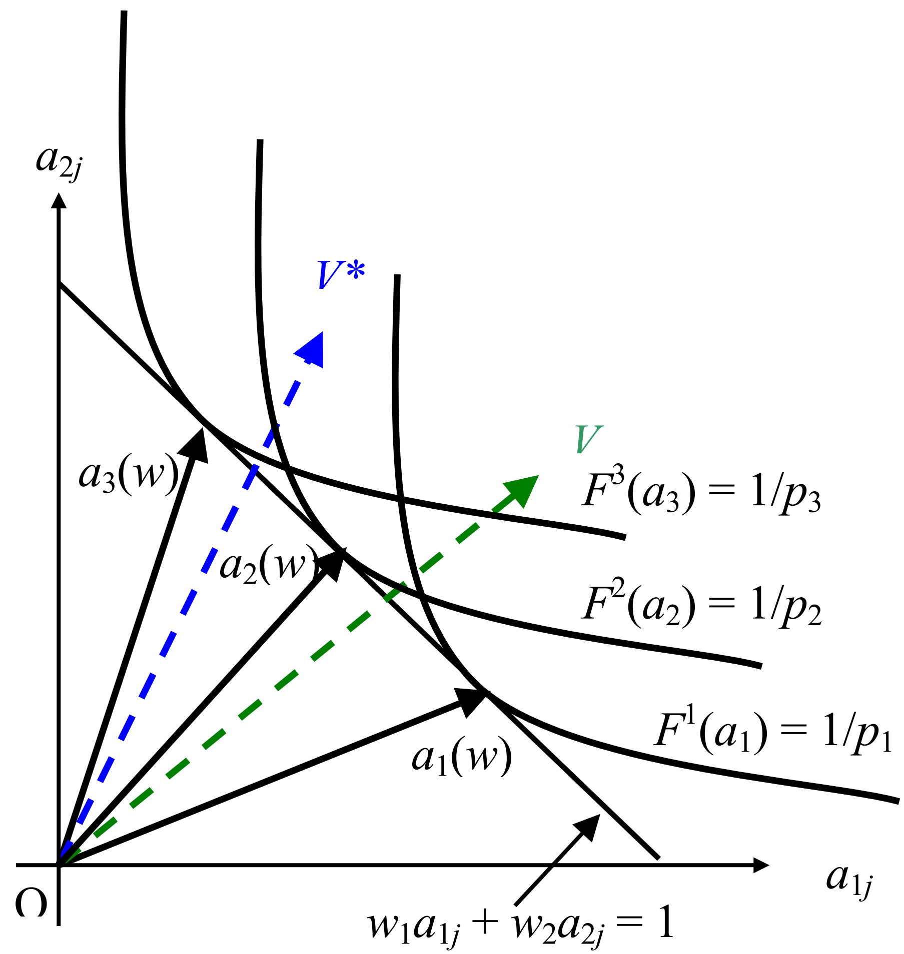
Consider the case of $M = 2 < N = 3$.

Assume No FIR such that

$$\frac{a_{11}(w)}{a_{21}(w)} > \frac{a_{12}(w)}{a_{22}(w)} > \frac{a_{13}(w)}{a_{23}(w)} \text{ for all } w.$$

Thus, sector-1 is more factor-1 intensive than sector-2, which is more factor-1 intensive than sector 3.

As long as the endowment vectors belong to the cone spanned by $a_1(w)$ and $a_3(w)$, the production vectors are indeterminate.



Heckscher-Ohlin Model with Many Goods without FPE

We have just seen that, with $N > M$ and with FPE, HO theorem fails. Paradoxically, a generalized version of the HO Theorem can be restored for $N > M = 2$, if FPE fails due to the dissimilarity of the factor endowments of the two countries.

A Chain of Comparative Advantage:

Since $R(p,V)$ is convex in V, $\dfrac{v_1}{v_2} > \dfrac{v_1^*}{v_2^*} \Rightarrow \omega \equiv \dfrac{w_1}{w_2} < \dfrac{w_1^*}{w_2^*} \equiv \omega^*$, whenever FPE fails.

From the Factor Intensity Assumption, $\dfrac{a_{1j}(w)}{a_{2j}(w)}$ is strictly decreasing in j for all w,

$$\Rightarrow \frac{C^j(w)}{C^k(w)} = \frac{C^j(\omega,1)}{C^k(\omega,1)} \text{ is strictly increasing in } \omega \text{ for } j < k.$$

$$\Rightarrow \frac{C^j(w)}{C^j(w^*)} = \frac{w_2 C^j(\omega,1)}{w_2^* C^j(\omega^*,1)} < \frac{w_2 C^k(\omega,1)}{w_2^* C^k(\omega^*,1)} = \frac{C^k(w)}{C^k(w^*)} \text{ for } j < k \text{ if } \omega < \omega^*.$$

Thus, there exists a marginal good, m, such that all the goods $j < m$ are produced only at Home, and all the goods $j > m$ are produced only at Foreign.

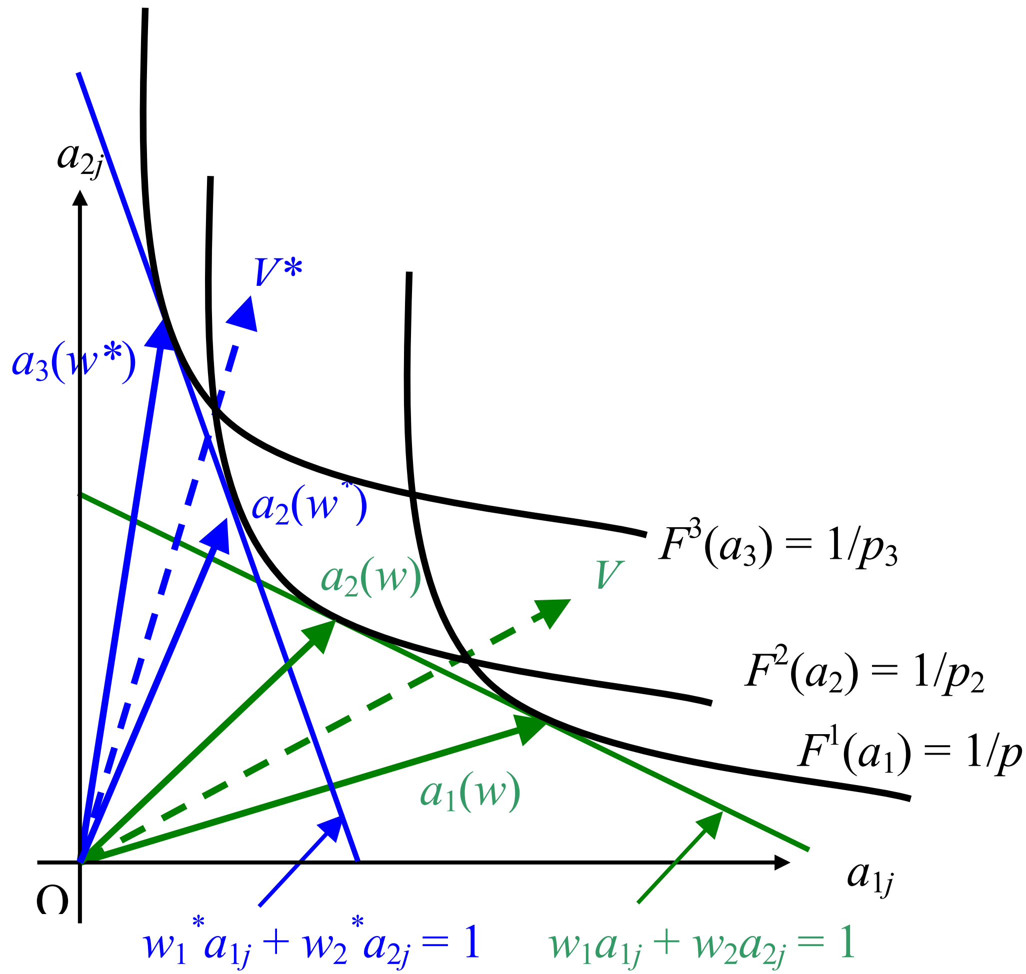
A Chain of Comparative Advantage: A Lerner Diagram Illustration

Good-1 is produced only by Home.
Good-3 is produced only by Foreign.
Good-2 is produced by both.

In order for this pattern to be an equilibrium, the endowment vector of each country (indicated by the dotted arrows) must belong to the cone spanned by the input vectors of two sectors in which each country produces. See Home (Green) and Foreign (Blue)

In this case,

Home exports the most factor-1 intensive Good 1 and Foreign exports the most factor-2 intensive Good 3.



In a nutshell,

The unequal factor prices yield a natural ordering of goods and a chain of comparative advantage based on factor intensities.

Heckscher-Ohlin-Vanek Theorem without FPE

Notice also that

- All production activities at Home are more factor-1 intensive technologies than all production activities at Foreign.

- Therefore, Home (Foreign) has to be the net export of factor-1 (2), when looking at Factor Content of Net Trade (as long as we calculate the factor content of any good by using the producer's input matrix.)

- Furthermore, this prediction holds even with FIR.

- More on HOV, see Feenstra (2003, Ch.3) and Davis and Weinstein (2003).

Heckscher-Ohlin-(Vanek) Theorem without FPE in a Multi-Country World

These observations can be extended to a multi-country setting.

Example: $M = 2 < C = 3 < N$.

- No FIR, and $\dfrac{a_{1j}(w)}{a_{2j}(w)}$ is strictly decreasing in *j* for all *w*.
- $\dfrac{v_1^c}{v_2^c}$ is strictly decreasing in *c,* so that, if FPE fails, $\dfrac{w_1^c}{w_2^c}$ is strictly increasing in *c*.

Then,

- The most factor-1 abundant Country 1 produces the most factor-1 intensive Good 1.
- The most factor-2 abundant Country 3 produces the most factor-2 intensive Good N.
- At most one good may be produced by Country 1 and 2.
- At most one good may be produced by Country 2 and 3.

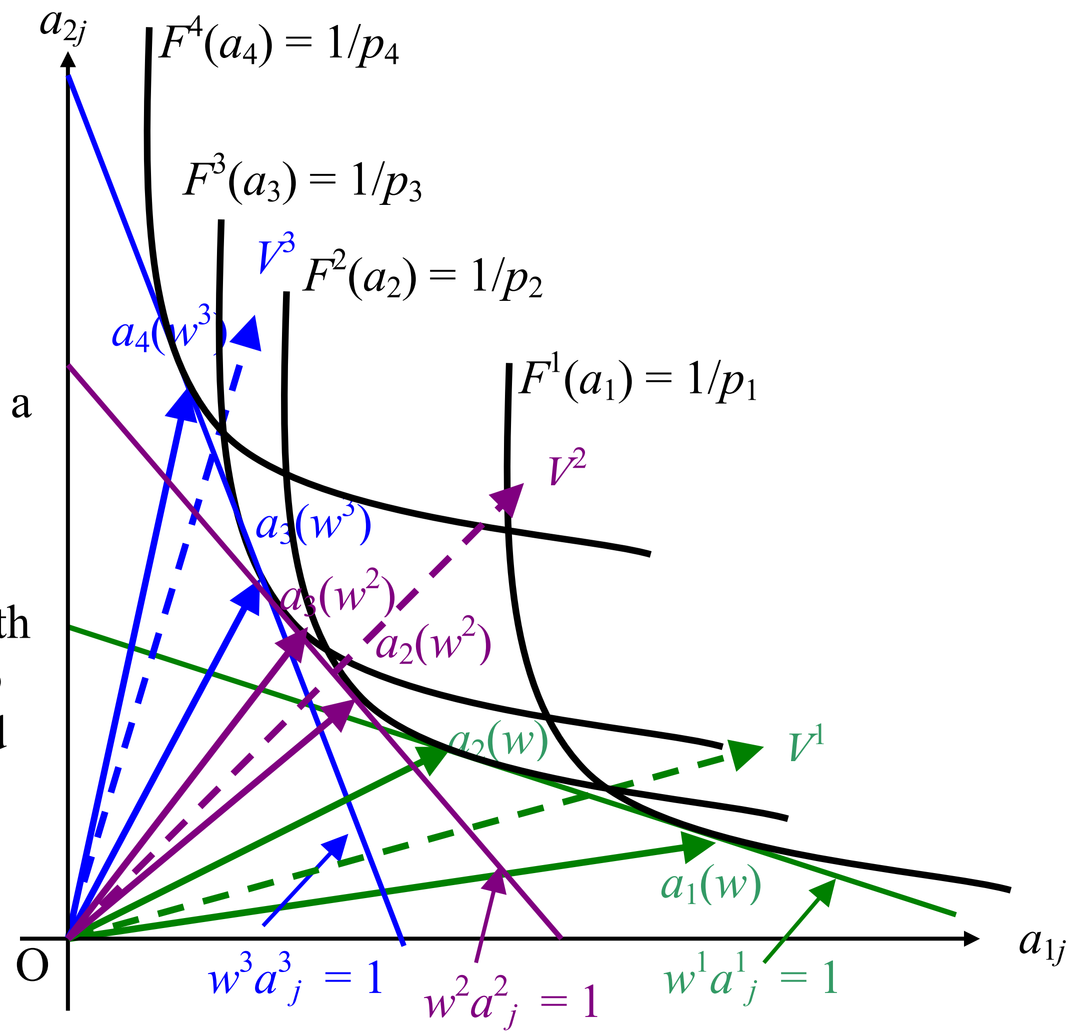
Figure illustrates one possible patterns for $N = 4$.

Country 1 (Green) produces 1 & 2.
Country 2 (Purple) produces 2 & 3.
Country 3 (Blue) produces 3 & 4.

This argument can be generalized to any number of countries.

Again, the unequal factor prices yield a natural ordering of both goods and countries

Recall that in the Ricardian model with many goods and countries, there is no natural way of ordering the goods and countries, since the patterns of comparative advantage are given exogenously.



Heckscher-Ohlin Model with a Finite Number of Goods and a Continuum of Countries ($M = 2 < N < \infty$; $C = [0, 1]$)

Assume

- No FIR and lower-indexed goods are more factor-1 intensive.
- Lower-indexed countries are more factor-1 abundant than higher-indexed countries.

The following configuration arises in equilibrium.

- Countries are partitioned into a finite set of intervals. Index each interval (that is, a group of countries) so that lower-indexed countries belong to the lower-indexed intervals.
- The number of intervals is at most equal to N.
- FPE holds within each interval, but not across intervals. (The number of intervals may be one, in which case FPE holds for the entire world.) The relative price of factor-1 is lower in the lower-indexed group.
- Lower-indexed goods are produced by lower-indexed groups of countries. At most one good is produced by two adjacent groups of countries.

This prediction, however, can be overturned in the presence of trade costs. See Deardorff (1979).

Heckscher-Ohlin Model with a Continuum of Goods (M = 2 < N = ∞): DFS (1980)

Two Factors (M = 2): $i = 1$ or 2.

A Continuum of Tradeable Goods: $j \in [0,1]$.

No Factor Intensity Reversal: $\dfrac{a_{1j}(w)}{a_{2j}(w)}$ is strictly decreasing in $j \in [0,1]$ for all w.

Two Countries: Home with $V = (v_1, v_2)$; Foreign(*) with $V^* = (v^*_1, v^*_2)$, where $\dfrac{v_1}{v_2} > \dfrac{v_1^*}{v_2^*}$.

Identical Cobb-Douglas Preferences:

$$\log U = \int_0^1 \alpha_j \log c_j dj \text{ and } \log U^* = \int_0^1 \alpha_j \log c_j^* dj \qquad \text{with } \int_0^1 \alpha_j dj = 1.$$

Equilibrium Conditions:

(PC): $p_j \le wa_j(w), p_j \le w^* a_j(w^*)$; $[p_j - wa_j(w)]x_j = [p_j^* - w^* a_j(w^*)]x_j^* = 0, j \in [0,1]$.

Home Factor Markets: $\int_0^1 a_{ij}(w) x_j dj = V_i,$ $\quad i = 1, 2.$

Foreign Factor Markets: $\int_0^1 a_{ij}(w^*) x_j^* dj = V_i^*$ $\quad i = 1, 2.$

Goods Markets: $p_j(x_j + x_j^*) = \alpha_j(wV + w^* V^*)$ $\quad j \in [0,1].$

Integrated Equilibrium:

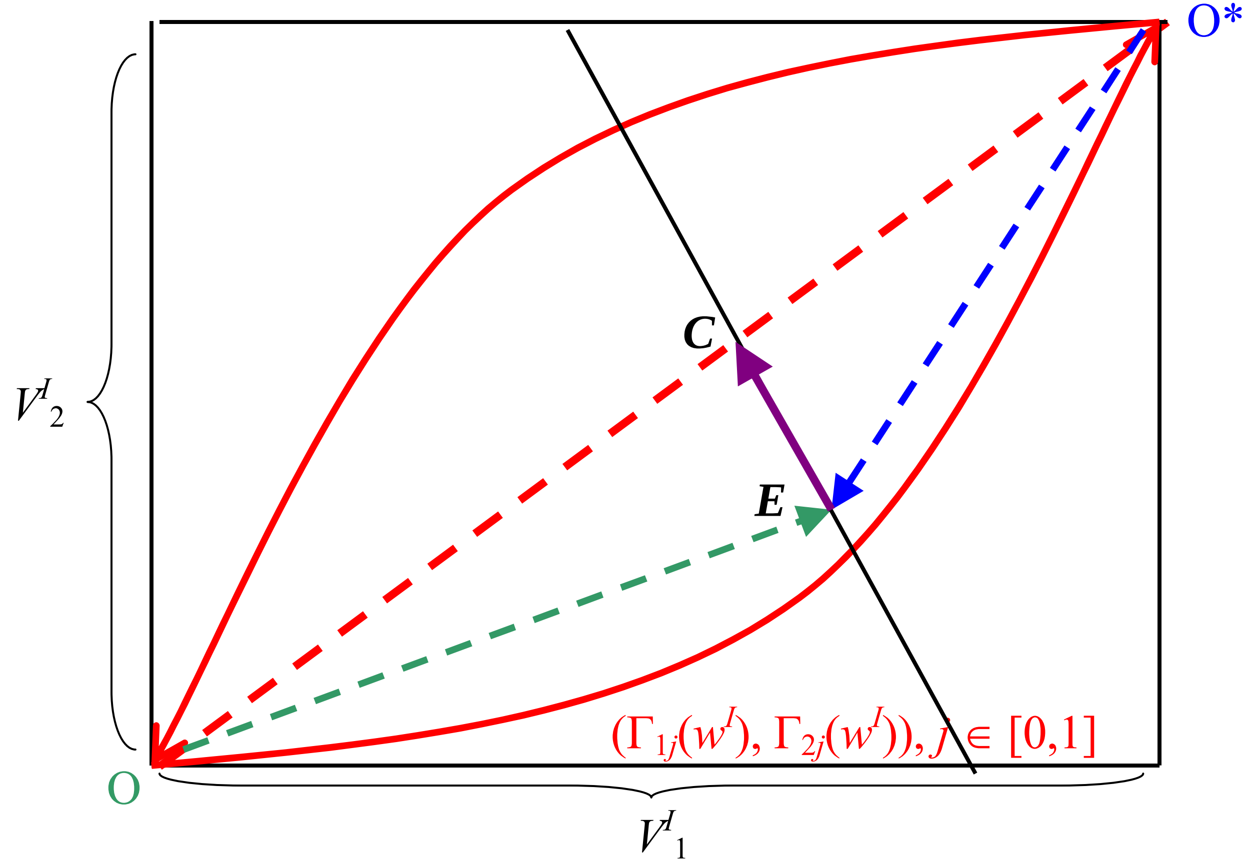
(PC): $p_j^I = w^I a_j(w^I),$ $\quad j \in [0,1].$

(RC): $\int_0^1 a_{ij}(w^I) x_j^I dj = V_i^I \equiv V_i + V_i^*,$ $\quad i = 1$ or $2.$

Goods Markets: $p_j^I x_j^I = \alpha_j w^I (V + V^*)$ $\quad j \in [0,1].$

Integrated Equilibrium and the FPE Set

- FPE holds if the division of the world endowment is inside the Red Lens, whose lower boundary is given by $(\Gamma_{1j}(w^I), \Gamma_{2j}(w^I)), j \in [0,1]$, where $\Gamma_{ij}(w) \equiv \int_0^j a_{ik}(w) x_k^I dk$.
- Purple Arrow depicts the Factor Content of Trade.
- Patterns of Production and Trade are indeterminate in the interior of the Red Lens. (The set of equilibrium shrinks in size continuously to zero, as E approaches the edge of the Red Lens).



What happens outside the FPE Set?

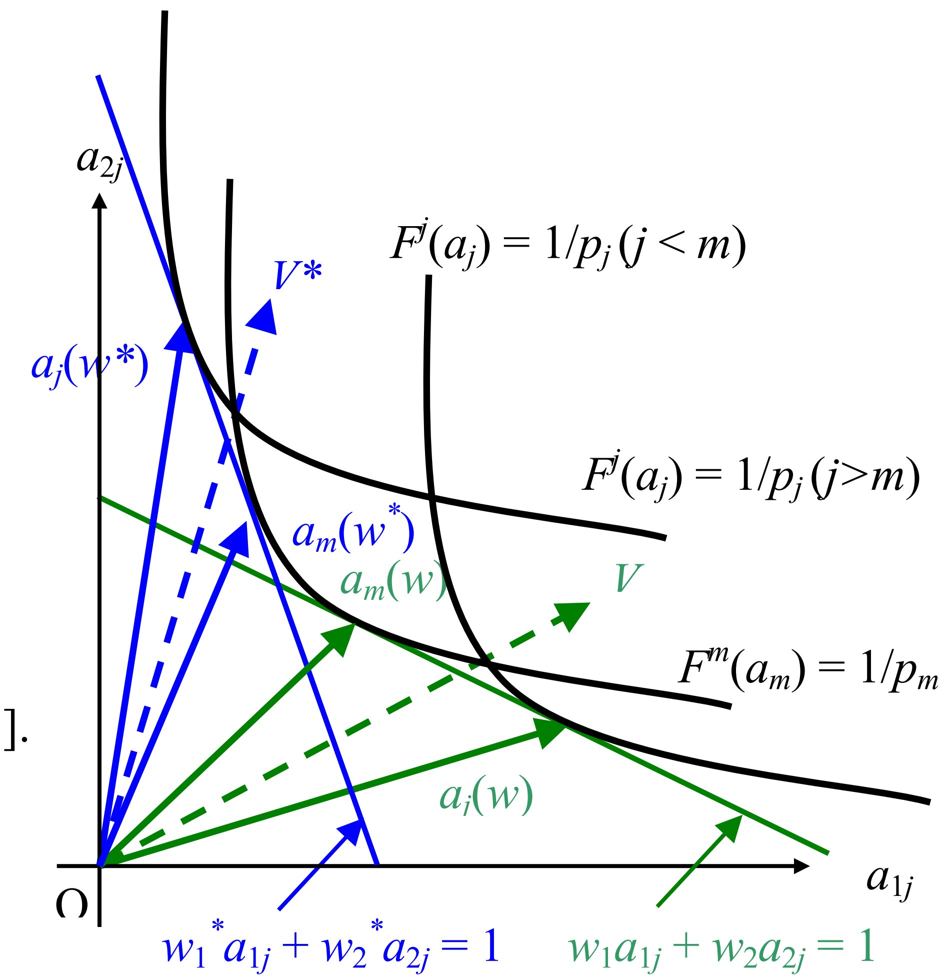
- FPE fails and $\frac{w_1}{w_2} < \frac{w_1^*}{w_2^*}$ $\Rightarrow$ A chain of comparative advantage based on Factor Intensities and Factor Endowments.

- There is a marginal sector, m, such that

Home produces all $j \in [0,m)$.
Foreign produces all $j \in (m,1]$.

Hence,

$$p_j = \begin{cases} C^j(w) = wa_j(w), & j \in [0,m). \\ C^j(w^*) = w^* a_j(w^*), & j \in (m,1]. \end{cases}$$



The marginal sector, *m,* and hence the patterns of trade are determined by

(PT): $$wa_m(w) = C^m(w) = C^m(w^*) = w^* a_m(w^*)$$

Factor Markets conditions are now $V_i = \int_0^m a_{ij}(w) x_j dj$; $V_i^* = \int_m^1 a_{ij}(w^*) x_j^* dj$ (i = 1, 2).
Or, using $p_j x_j = \alpha_j (wV + w^* V^*)$ for $j \in [0,m)$; $p_j x_j^* = \alpha_j (wV + w^* V^*)$ for $j \in (m, 1]$,

(FM): $$\frac{w_i V_i}{wV + w^* V^*} = \int_0^m \theta_{ij}(w) \alpha_j dj\,; \quad \frac{w_i^* V_i^*}{wV + w^* V^*} = \int_m^1 \theta_{ij}(w^*) \alpha_j dj\,; \qquad (i = 1, 2).$$

where $\theta_{ij}(\bullet)$ is the factor-i's share in the cost of sector-j, and $\theta_{1j}(w) + \theta_{2j}(w) = 1$. Hence, adding up (FM) for either country yields the Balanced Trade Condition:

(BT): Home Imports = $wV\left[1 - \int_0^m \alpha_j dj\right] = w^* V^* \int_0^m \alpha_j dj$ = Foreign Imports

- Four independent equilibrium conditions.
- Four endogenous variables, m, and $2M-1$ = 3 relative factor prices.

In fact, we can further simplify the system by denoting the relative factor prices by $\omega \equiv w_1/w_2$ and $\omega^* \equiv w_1^*/w_2^*$ and noting that $a_m(\bullet)$ & $\theta_{ij}(\bullet)$ both satisfy the homogeneity of degree zero, so that they depend solely on the relative factor price.

(FM) can now be rewritten to

Home Factor Market Eq.: $$\omega\frac{V_1}{V_2} = \frac{\int_0^m \theta_{1j}(\omega)\alpha_j dj}{\int_0^m \theta_{2j}(\omega)\alpha_j dj}$$

Foreign Factor Market Eq: $$\omega^*\frac{V_1^*}{V_2^*} = \frac{\int_m^1 \theta_{1j}(\omega^*)\alpha_j dj}{\int_m^1 \theta_{2j}(\omega^*)\alpha_j dj}$$

(PT) and (BT) may be combined into:

(PT)+(BT): $$\frac{\omega V_1 + V_2}{\omega^* V_1^* + V_2^*}\frac{\omega a_{1m}(\omega) + a_{2m}(\omega)}{\omega^* a_{1m}(\omega^*) + a_{2m}(\omega^*)} = \frac{\int_0^m \alpha_j dj}{1 - \int_0^m \alpha_j dj}.$$

This system of 3 equations determines 3 endogenous variables, ω, ω^* and m.

Exercise: Suppose that the production functions are all Cobb-Douglas, so that $\theta_{1j} = 1 - \theta_{2j}$ is independent of ω (and strictly decreasing in j).

1) Show that $\omega^A < \omega \leq \omega^I \leq \omega^* < \omega^{*A}$.

2) Show the condition under which $\omega < \omega^I < \omega^*$.

3) Characterize the equilibrium under the above condition.

4) Evaluate the gains from trade.

5) Under the above condition, show that a proportional increase in V_1 and V_2, which holds V_1/V_2 constant, would lead to a decline in ω and ω^* and an increase in m.

6) Evaluate the welfare effects of the above change.

7) Consider a multi-country extension of this model, where V_1^c / V_2^c is strictly decreasing in c, where $c = 1, 2, \ldots, C$. Write down the equilibrium conditions under the assumption that the factor prices are not equalized.

What is the intuition behind result in 5)? For the sake of concreteness, let us suppose

- Home is unskilled labor-abundant South, Foreign is skilled labor abundant North.
- Factor-1 is unskilled labor; and Factor-2; skilled labor.

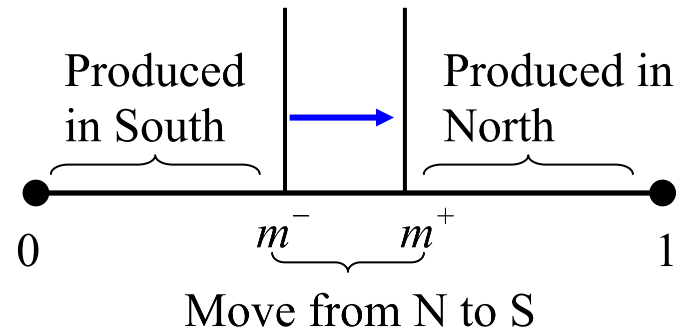
Then, without FPE,

- South's skilled premium is larger than in North ($\omega < \omega^*$).
- South specializes in lower-range of goods that are less skill intensive, $j \in [0,m)$.
- North specializes in higher-range of goods that are more skill intensive, $j \in (m,1]$

Now, suppose that South grows in its (relative) size, in a way that keeps its factor proportion unchanged: $\hat{v}_1 = \hat{v}_2 > 0$. This may occur for different reasons.

- Population may grow faster in the South than in the North.
- The South experiences faster (uniform) productivity growth than the North, which may be due to technology diffusion from North to South. (Recall that factor endowments may be measured in their effective units.)
- More countries in the South may join the global economy.

As South grows in its relative size, it produces a wider range of goods to keep its factors employed. (Its total expenditure goes up as well, but it is spread across all goods.) Hence, m goes up for m^- to $m^+ > m^-$.



- The industries that moved to South, $j \in (m^-, m^+)$, are the most skill-intensive of all industries in the South. The South's labor demand composition shifts toward skilled labor, causing the skill premium to rise in the South.
- The industries that leave North, $j \in (m^-, m^+)$, are the least skill-intensive of all industries in the North. They specialize even more skill-intensive goods. The North's labor demand composition shifts toward skilled labor, causing the skill premium to rise in the North.

Notes:

- Viewed this way, the global rise in the skill premium may be due to globalization.
- In this experiment, the world relative supply, $(v_1 + v^*_1)/(v_2 + v^*_2)$, goes up, even though the relative supply in each country, $v_1/v_2 > v^*_1/v^*_2$, is unchanged, because the relative size of the unskilled-labor abundant country goes up.
- More on the skill premium and trade, start with Feenstra (2004, Ch.4).

Trade Costs in a Heckscher-Ohlin Model with a Continuum of Goods

We have just seen that the dissimilarity of the factor proportions can cause FPE to fail, which restores the determinacy of the patterns of production and trade, and the HO-theorem.

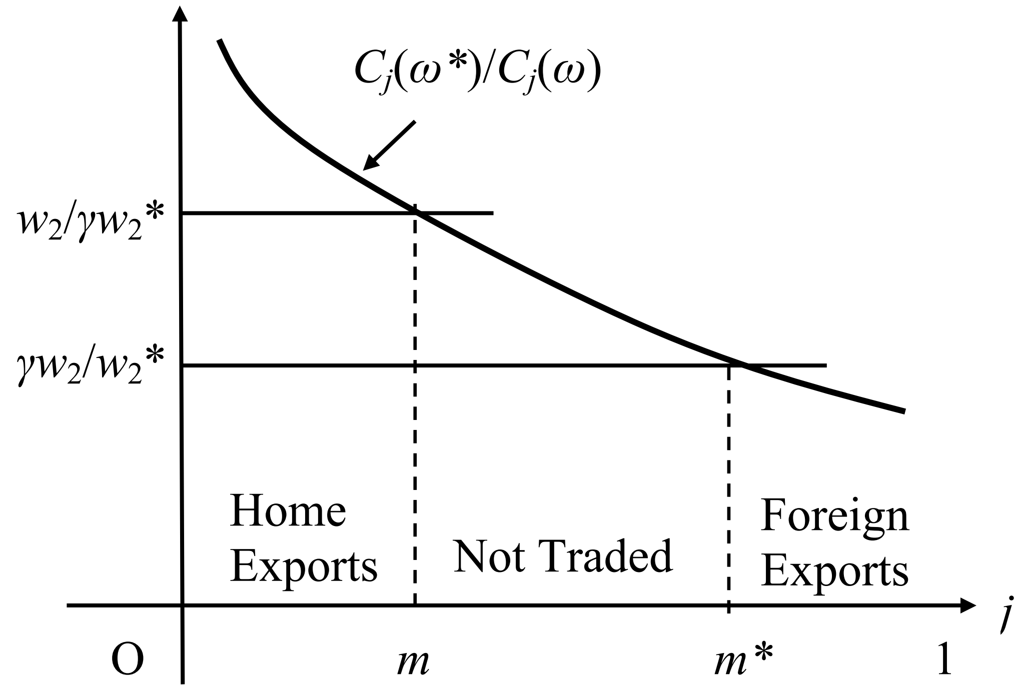
The presence of trade costs is another way in which FPE fails and the HO-theorem may be restored.

Let us introduce the iceberg cost into the HO model of DFS (1980). For one unit of each good to arrive to the destination, $1/\gamma > 1$ units of the good must be shipped.

Then,

- Home exports good-j to Foreign if $C^j(w) = wa_j(w) < \gamma C^j(w^*) = \gamma w^* a_j(w^*)$
- Foreign exports good-j to Home if $\gamma C^j(w) = \gamma wa_j(w) > C^j(w^*) = w^* a_j(w^*)$
- Otherwise, goods are not traded.

Exercises:



Let $C_j(\omega) \equiv C_j(\omega, 1)$.

- Show that, if $\omega < \omega^*$, $C_j(\omega^*)/C_j(\omega)$ is strictly decreasing in j, as shown in the figure.
- Show that, if $\omega < \omega^*$, the goods are divided into three intervals, as shown in the figure, as long as γ is not too small.
- Show that $\omega < \omega^*$ in equilibrium.
- Derive the system of five equations that determines ω, ω^*, w_2/w^*_2, m, and m^*.
- Assume that Cobb-Douglas preferences are symmetric; $\alpha_j = 1$. Let $v_1 = v^*_2 = 1 - v^*_1 = 1 - v_2 = v > ½$, and $\theta_{1j} = 1 - \theta_{2j} = 1 - j$. Examine the effects of changing v and γ.

Note: Since Home and Foreign producers face different output prices, the convexity of $R(p, V)$ in V cannot be used to prove $\omega < \omega^*$.

Non-homothetic Preferences in a Heckscher-Ohlin Model with a Continuum of Goods (Unfinished)

Notice that, without FPE, the Factor Proportion Model with $M = 2 < N$ is similar to the Ricardian model in many ways. In particular,

- A Chain of Comparative Advantage based on factor intensities
- Extreme patterns of production, where many goods are produced in only one country.

This is partly due to the assumption that all factors are perfectly mobile. We now look at some models with partially mobile factors.

Models with Partially Specific or Partially Mobile Factors

- Both the Heckscher-Ohlin and Specific Factors Models make the extreme assumption: that all factors are either
  - not specific to any sector and hence perfectly mobile
  - completely specific to a particular sector and hence immobile.

- The truth probably lies in between. There have been some attempts to introduce partially specific (or partially mobile) factors. For example,

- In an otherwise standard Ricardian model, Ruffin (1988) and Matsuyama (1992) introduced heterogeneous workers, so that not every worker switch sectors simultaneously in response to a change in the relative wage rates across sectors.

- Starting from the standard 2-Sector Specific Factor Model, with 2 (completely) specific capital and one (completely) mobile labor,
  - Mussa (1982, sec. 4) makes labor less than perfectly mobile by introducing heterogeneous workers, while keeping the complete specificity of capital.
  - Grossman (1983) makes capital partially mobile by introducing heterogeneous capital, while keeping the complete mobility of labor.

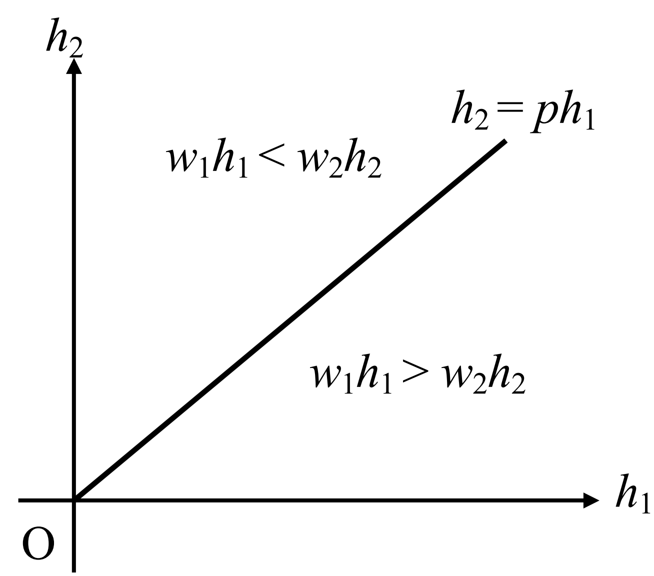
A Ricardian Model with Heterogeneous Workers: Based on Matsuyama (1992)

- **A Two-Sector Ricardian Model,** with $a_1 = a_2 = 1$. That is, one unit of the effective labor produces one unit of the good in each sector. Then, the wage rates are given by $w_1 = p_1$ and $w_2 = p_2$.
- **Heterogeneous Workers,** with the characteristics**,** $h = (h_1, h_2)$, where h_j is the unit of effective labor services a worker can supply when allocated to sector-j (j =1, 2). Let $G(h)$ be the cdf with the density, $g(h) > 0$.
- All workers with $w_1h_1 > w_2h_2$ choose sector-1 and all workers with $w_1h_1 < w_2h_2$ choose sector-2, while all workers with $w_1h_1 = w_2h_2$ are indifferent.
- The Total Labor Supply and Output in each sector are thus given by

$$X_1(p) = L_1(p) = \int_0^{\infty}\int_0^{ph_1} h_1 g(h_1, h_2)dh_2 dh_1$$

$$X_2(p) = L_2(p) = \int_0^{\infty}\int_{ph_1}^{\infty} h_2 g(h_1, h_2)dh_2 dh_1$$

where $p \equiv p_1/p_2$.



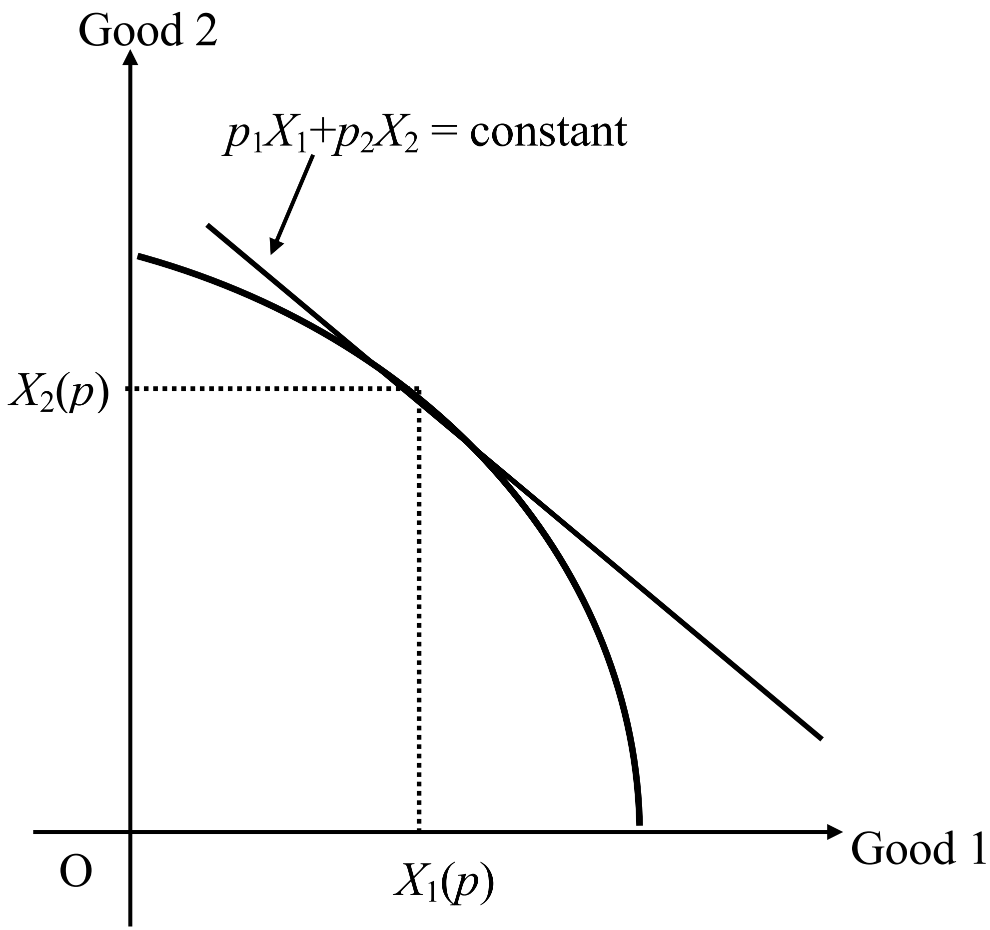
Exercise: Show that PPF of this economy is strictly convex, and its slope at the equilibrium production is given by the relative price, $p = p_1/p_2$.

Notes:

- This model may be viewed as a MxN factor proportions model, where N = 2 < M = ∞, the opposite of the HO model with a continuum of goods.

- In this model, individual workers specialize, following the law of comparative advantage.

- The patterns of trade and factor prices are determined by the factor proportions (i.e., the distribution of the labor types), just as any other factor proportion models.



- The standard two-sector Ricardian model may be viewed as a special case of this factor proportion theory, where Home has a degenerate factor distribution, $h = (1/a_1, 1/a_2)$ for all workers, and Foreign has a degenerate factor distribution, $h^* = (1/a_1^*, 1/a_2^*)$. The two interpretations are identical, as long as we maintain the assumption that all labor are immobile across countries.

- The above example can easily be generalized to the case of N-tradeable goods sectors by making $h = (h_1, \ldots, h_N)$, distributed according to $G(h)$. But, how can we make such a multi-sector model operational?

- Instead of taking $G(h)$ as the primitive, Ohnsorge-Trefler (2007) proposed:

➢ The effective labor an individual worker can supply to sector-j, h_j, is a Cobb-Douglas function of the two *attributes or skills* s/he has, H and L, $h_j = T(H, L, j) = H^{\beta(j)} L^{1-\beta(j)}$, where $\beta(j)$ is strictly increasing in j. Then, the workers are sorted out according to the ratio, H/L; a worker with a higher H/L goes to a higher-indexed sector.

➢ The distribution of H and L is a log-normal.

Using this model, they showed how the *higher* moments of the skill distributions can be interesting predictors of trade and examined the impact of trade on income distributions.

***Example, inspired by Ohnsorge and Trefler* (*2007*)**.

Consider a 3-sector model with Home and Foreign.

Technologies: Each sector transforms one unit of effective labor into one unit of the output. A worker with attributes, (*H, L)*, can supply

- H units of effective labor in Sector 1.
- $H^{\beta}L^{1-\beta}$ units of effective labor in Sector 2.
- L units of effective labor in Sector 3.

Distribution of Workers: Each country is populated by a unit measure of workers.

- At Home, workers are homogeneous, with (*H, L*) = (1, 1).
- At Foreign, 50% of workers have (*H, L*) = (2, 0); 50% of workers have (*H, L*) = (0, 2).

Exercise: Predict the patterns of trade in this example.

Note: It is essential that, in this model,

- Each worker comes as a "package deal." A worker with $(H, L) = (1, 1)$ cannot supply one unit of H and one unit of L separately. (He could part a fraction of his time in different sectors, but he could supply H and L only in the same proportion in each sector.)
- Workers with different attributes do not complement each other. Hiring one worker with $(H, L) = (2, 0)$ and one worker with $(H, L) = (0, 2)$ cannot duplicate what two workers with $(H, L) = (1, 1)$ can do.

Examples:

- Ohnsorge-Trefler discuss IT industry having difficulty finding employees who have the right mix of skills, technically capable and good in communicating skills.
- To succeed as an academic economist, you need to be good at *both* solving models *and* writing papers.

Alternative Approaches to Introducing Partial Factor Mobility:

Dynamic Models with Short-Run Specificity

- Mussa (1978)
- Neary (1978)
- Matsuyama (1991, 1992)

We will discuss these models in Part V.

A Model with Lumpy Countries

Courant and Deardorff ??

A Model with Endogenous Specificity

Grossman and Shapiro (1982)??

Models with Search and Informational Mobility Frictions
??

Factor Proportion Models with Technological Heterogeneities

- Davis (1995): Factor Intensity Differ Across Sectors plus Technological Heterogeneities within Sectors

- Zhu and Trefler (2005): Technological Heterogeneities correlated with Factor Intensity Differences Across Sectors

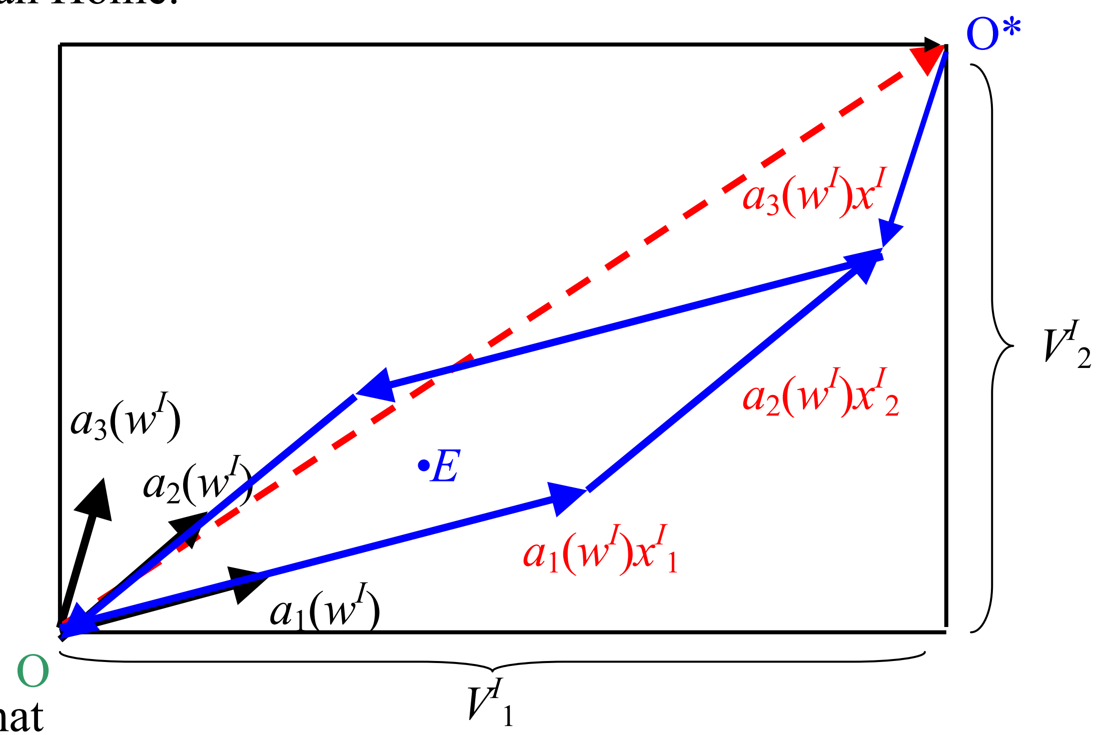
- Matsuyama (2007): Technological Heterogeneities across goods, Factor Intensity Differ Across Destinations

Davis and Zhu-Trefler built a model where both Technological (Ricardian) and Factor-Endowment (Heckscher-Ohlin) reasons for trade co-exist.

In Matsuyama (2007), the basis for trade is entirely Ricardian. However, factor intensities and factor proportions matter because they affect trade costs. The reason is that factor intensities depend on the destination of the goods.

A Ricardo-Heckscher-Ohlin Model with a Finite Number of Goods

- Consider the case of 2-countries, Home & Foreign, and 3-goods, 2-factors.
- Home and Foreign share the identical technologies in producing Goods 1 & 2, but Foreign has higher TFP for Good 3 than Home.
- In Integrated Equilibrium (IE), where factors are mobile but not technologies, Good 3 is produced only at Foreign.
- Now, impose the factor immobility. To replicate IE and FPE, Good 3 must be produced at Foreign, which requires E to be in the Blue Box.
- Inside the Blue Box, we may proceed to analyze the patterns and the volume of trade, etc.



Davis (1995) made further assumption that Good 3 and Good 2 have the same factor intensity, belonging to the same industry to address the issues of intra versus inter-industry trade.

A Ricardo-Heckscher-Ohlin Model with a Continuum of Goods (Unfinished)
/
Two Countries: Home and Foreign(*)

***M*-factors:** indexed by $i = 1, 2, ..., M$.

Endowments: M-dimensional Column Vectors

$V = (V_1, \ldots, V_M)^T$, $\quad V^* = (V_1^*, \ldots, V_M^*)^T$

Factor Prices: M-dimensional Row Vectors

$w = (w_1, \ldots, w_M)$, $\quad w^* = (w_1^*, \ldots, w_M^*)$

***J*-Sectors,** indexed by $j = 1, 2, ..., J$.

Each sector produces *a continuum of goods,* indexed by $z_j \in [0,1]$.

Cobb-Douglas Preferences:

$$U = \sum_{j=1}^{J} \alpha_j \int_0^1 \log x(z_j) dz_j, \; U^* = \sum_{j=1}^{J} \alpha_j \int_0^1 \log x^*(z_j) dz_j, \text{ where } \sum_{j=1}^{J} \alpha_j = 1.$$

CRS Technologies:

Unit Cost Function of good z_j in Sector-j: $a_j(z_j)\Phi_j(w)$ and $a_j^*(z_j)\Phi_j(w^*)$,

(A1) $A_j(z_j) \equiv a_j^*(z_j)/a_j(z_j)$ is continuous and decreasing in z_j.
(A2) $\Phi_j(\bullet)$ is linear homogeneous, increasing, concave.

Notes: The model is Ricardian within each sector, HO across sectors

- All goods in Home and Foreign share the same Φ_j.
- Technological differences between Home and Foreign are only in TFP.
- Φ_j does not depend on z_j. (All goods that belong to the same sector have the identical factor intensities at any factor prices).

Patterns of Trade:

$$p(z_j) = \min\{a_j(z_j)\Phi_j(w),\ a_j^*(z_j)\Phi_j(w^*)\} = \begin{cases} a_j(z_j)\Phi_j(w) & \text{if } z_j < m_j \\ a^*_j(z_j)\Phi_j(w^*) & \text{if } z_j > m_j \end{cases}$$

where $A_j(m_j) = \dfrac{\Phi_j(w)}{\Phi_j(w^*)}$

Factor Market Equilibriums:

Demand for Home Factor-i by the good produced in the Home sector-j, $z_j < m_j$:

$$a_j(z_j)\frac{\partial \Phi_j(w)}{\partial w_i}D(z_j) = \frac{p(z_j)}{\Phi_j(w)}\frac{\partial \Phi_j(w)}{\partial w_i}D(z_j) = \frac{\partial \log \Phi_j(w)}{\partial w_i}\alpha_j(wV + w^* V^*)$$

Demand for Home Factor-i in Sector-j:

$$\frac{\partial \log \Phi_j(w)}{\partial w_i}\alpha_j m_j(wV + w^* V^*)$$

Home Factor-i Equilibrium:

$$V_i = (wV + w^* V^*)\sum_{j=1}^{J}\frac{\partial \log \Phi_j(w)}{\partial w_i}\alpha_j m_j$$

Market Equilibrium for Home Factor-i:

$$w_i V_i = (wV + w^* V^*)\sum_{j=1}^{J}\theta_{ij}(w)\alpha_j m_j\,,$$

$\theta_{ij}(w) \equiv \dfrac{\partial \log \Phi_j(w)}{\partial \log(w_i)}$: the factor-i's share in sector-j, with $\sum_{i=1}^{M}\theta_{ij}(w) = 1$.

Similarly, *Market Equilibrium for Foreign Factor-i:*

$$w_i^* V_i^* = (wV + w^* V^*)\sum_{j=1}^{J}\theta_{ij}(w^*)\alpha_j(1 - m_j)\,.$$

Equilibrium Conditions:

(PT) $$A_j(m_j) = \frac{\Phi_j(w)}{\Phi_j(w^*)} \qquad (j = 1, 2,\ldots, J)$$

(Home FM) $$w_i V_i = (wV + w^* V^*)\sum_{j=1}^{J} \theta_{ij}(w)\alpha_j m_j \qquad (i = 1, 2,\ldots, M)$$

(Foreign FM) $$w_i^* V_i^* = (wV + w^* V^*)\sum_{j=1}^{J} \theta_{ij}(w^*)\alpha_j (1 - m_j) \qquad (i = 1, 2,\ldots, M)$$

Adding up the factor market conditions for all i in either country yields

(BT): $$wV\sum_{j=1}^{J} \alpha_j (1 - m_j) = w^* V^* \sum_{j=1}^{J} \alpha_j m_j$$

- 2M + J − 1 Equilibrium Conditions
- 2M + J − 1 Unknowns (J marginal goods and 2M − 1 relative factor prices)

Define $m = \sum_{j=1}^{J} \alpha_j m_j$.

Equilibrium Conditions:

(PT): $$m = \sum_{j=1}^{J} \alpha_j A_j^{-1}\left(\frac{\Phi_j(w)}{\Phi_j(w^*)}\right)$$

(H-FM): $$w_i V_i = (wV + w^* V^*)\sum_{j=1}^{J} \theta_{ij}(w)\alpha_j A_j^{-1}\left(\frac{\Phi_j(w)}{\Phi_j(w^*)}\right) \qquad (i = 1,2,\ldots, M)$$

(F-FM): $$w_i^* V_i^* = (wV + w^* V^*)\sum_{j=1}^{J} \theta_{ij}(w^*)\alpha_j \left(1 - A_j^{-1}\left(\frac{\Phi_j(w)}{\Phi_j(w^*)}\right)\right) \qquad (i = 1,2,\ldots, M)$$

(BT): $$wV(1-m) = w^* V^* m.$$

Recovering DFS Ricardian Model-I: $\Phi_j(\bullet) = \Phi(\bullet)$ for all j → $\theta_{ij}(\bullet) = \theta_i(\bullet)$ for all j.

(PT): $$m = \sum_{j=1}^{J} \alpha_j A_j^{-1}\left(\frac{\Phi(w)}{\Phi(w^*)}\right)$$

(Home FM): $$\frac{w_i V_i}{\theta_i(w)} = m(wV + w^* V^*) \qquad (i = 1, 2, \ldots, M)$$

(Foreign FM): $$\frac{w_i^* V_i^*}{\theta_i(w^*)} = (1-m)(wV + w^* V^*) \qquad (i = 1, 2, \ldots, M)$$

(BT): $$wV(1-m) = w^* V^* m.$$

Aggregation Theorem: Define $\mathrm{F}(x) \equiv \min_q \{qx \mid \Phi(q) \geq 1\}$. Then,

$$wV = \Phi(w)\mathrm{F}(V) = WL, \text{ and } w^*V^* = \Phi(w^*)\mathrm{F}(V^*) = W^*L^*,$$

where
- $W \equiv \Phi(w)$: the Home factor price index, "the Home wage rate"
- $L \equiv \mathrm{F}(V)$: the Home factor quantity index, "Home labor."
- $W^* \equiv \Phi(w^*)$: "Foreign wage rate"
- $L^* \equiv \mathrm{F}(V^*)$: "Foreign labor."

(PT): $$m = \sum_{j=1}^{J} \alpha_j A_j^{-1}\left(\frac{W}{W^*}\right) = \sum_{j=1}^{J} \alpha_j A_j^{-1}(\omega)$$

(BT): $$\frac{W}{W^*} \equiv \omega = \frac{m}{1-m}\frac{L^*}{L}$$

(FM): $$\frac{w_i V_i}{m\theta_i(w)} = WL + W^*L^* = \frac{w_i^* V_i^*}{(1-m)\theta_i(w^*)} \qquad (i = 1, 2, \ldots, M)$$

- Independently of (FM), m and ω are solely determined by (PT) and (BT), which are essentially equivalent to those in the DFS Ricardian model.
- The factor proportion affects the factor prices via (FM) but has no effect on trade.

Recovering DFS Ricardian Model-II: $V^* = \mu V$ and $A_j(\bullet) = A(\bullet)$.

Then, $w = \eta w^*$ and

(PT): $$A(m_j) = A(m) = \eta$$

(BT): $$\eta = \frac{m}{1-m}\mu.$$

(FM): $$\frac{w_i V_i}{wV} = \frac{w_i^* V_i^*}{w^* V^*} = \frac{\eta+\mu}{\eta/m}\sum_{j=1}^{J}\theta_{ij}(w)\alpha_j = \frac{\eta+\mu}{\mu/(1-m)}\sum_{j=1}^{J}\theta_{ij}(w)\alpha_j$$

$(i = 1, 2, \ldots, M)$

Again, independently of (FM), m and η are solely determined by (PT) and (BT), which are essentially equivalent to those in the DFS Ricardian model.

This framework contains other models as special cases:

- ***MxJ Heckscher-Ohlin Model:*** $A_j(\bullet) = 1$ for all j.

- ***Davis* (1995) *Model:*** $J = M$ and $A_j(z_j) = 1$ for a positive measure of $z_j \in [0,1]$ and $A_j(\bullet) > 1$ for the rest.

If we allow for a continuum of sectors, so that $j \in [0,1]$,

- ***DFS Heckscher-Ohlin Model:*** $M = 2$ and $A_j(\bullet) = 1$ for all $j \in [0,1]$.

- ***Zhu-Trefler* (2005) *Model:*** $M = 2$ and $A_j(\bullet) = A_j$ is monotone in $j \in [0,1]$.

Here, let us try a different special case.

Let $M = J = 2$ and suppose that $A_j(\bullet) = A(\bullet)$ is strictly decreasing. Assume Cobb-Douglas so that $\theta_{ij}(w) = \theta_{ij}$ is constant. Without loss of generality, $1-\theta_{21} = \theta_{11} > \theta_{12} = 1-\theta_{22}$ (Sector-1 is more factor-1 intensive).

(PT) $$A(m_j) = \left(\frac{\omega}{\omega^*}\right)^{\theta_{1j}} \left(\frac{w_2}{w_2^*}\right) \qquad (j = 1, 2)$$

(FM) $$\omega \frac{V_1}{V_2} = \frac{\sum_{j=1}^{2} \theta_{1j}\alpha_j m_j}{\sum_{j=1}^{2} (1-\theta_{1j})\alpha_j m_j} \quad ; \quad \omega^* \frac{V_1^*}{V_2^*} = \frac{\sum_{j=1}^{2} \theta_{1j}\alpha_j (1-m_j)}{\sum_{j=1}^{2} (1-\theta_{1j})\alpha_j (1-m_j)}$$

(BT): $$\frac{wV}{w^*V^*} = \frac{w_2}{w_2^*} \frac{\omega(V_1/V_2)+1}{\omega^*(V_1^*/V_2^*)+1} = \frac{\sum_{j=1}^{2} \alpha_j m_j}{\sum_{j=1}^{2} \alpha_j (1-m_j)}$$

These 5 equations determine 5 unknowns; $\omega \equiv \frac{w_1}{w_2}$, $\omega^* \equiv \frac{w_1^*}{w_2^*}$, $\frac{w_2}{w_2^*}$, m_1, and m_2.

Continue…

Matsuyama (2007): Beyond Icebergs: Toward A Theory of Biased Globalization

Key Motivations:

- International business activities require more intensive use of certain factors than domestic business activities:
  - ✓Language skills
  - ✓Maritime Insurance
  - ✓Transoceanic transport is more capital intensive than local transport
- We need to go beyond the iceberg approach to model trade costs.

Framework: A DFS Ricardian model with a continuum of goods extended to allow for

- Multiple Factors
- Factor intensities may differ across destinations (but not across goods)

Main Results:

- Generalize the iceberg approach as the trade costs depends on the factor prices.
- Globalization changes the relative factor demands (and hence relative factor prices) in the same direction across countries (in contrast to Stolper-Samuelson) with some implications on the globalization/skill premium debate.
- The factor proportions (not the difference in the factor proportions) determines the volume of trade (in contrast to Heckscher-Ohlin).

Two Countries: Home and Foreign(*)

A Continuum of Goods; z ∈ [0,1].

Cobb-Douglas Preferences:

$$U = \int_0^1 b(z) \log C(z) dz; \qquad U^* = \int_0^1 b^*(z) \log C^*(z) dz$$

where $B(z) \equiv \int_0^z b(s)ds$; $B^*(z) \equiv \int_0^z b^*(s)ds$; with $B(0) = B^*(0) = 0$ and $B(1) = B^*(1) = 1$.

Two Key Departures from DFS (1977)

- Multiple Factors
- Destination Dependent Technologies (Differential Factor Intensities)

#1: J-Factors:

Endowments: J-dimensional Column Vectors

$V = (V_1, V_2, \ldots, V_J)^T \qquad V^* = (V_1^*, V_2^*, \ldots, V_J^*)^T$

Factor Prices: J-dimensional Row Vectors

$w = (w_1, w_2, \ldots, w_J) \qquad w^* = (w_1^*, w_2^*, \ldots, w_J^*)$

#2: CRS Technologies:

Unit Cost Functions		Destination	
		Home	Foreign
Origin	Home	$a(z)\Phi(w)$	$a(z)\Psi(w;\tau)$
	Foreign	$a^*(z)\Psi^*(w^*; \tau^*)$	$a^*(z)\Phi^*(w^*)$

- Φ and Ψ (Φ^* and Ψ^*) linear homogeneous, increasing, and concave.
- τ, τ^*; Shift Parameters (Export Technologies, or Sometimes Trade Barriers)
- TFP differ across Origins, but *not* across Destinations
- Factor Intensities differ across Destinations, but *not* across Industries

(A1) $A(z) \equiv a^*(z)/a(z)$ is continuous and strictly decreasing in z.

(A2) $\Phi(w) < \Psi(w;\tau)$; $\Phi^*(w^*) < \Psi^*(w^*;\tau^*)$.

Supplying (i.e., producing, marketing, communicating, shipping, insuring etc.) goods to the export market is costlier than supplying the same goods to the domestic market.

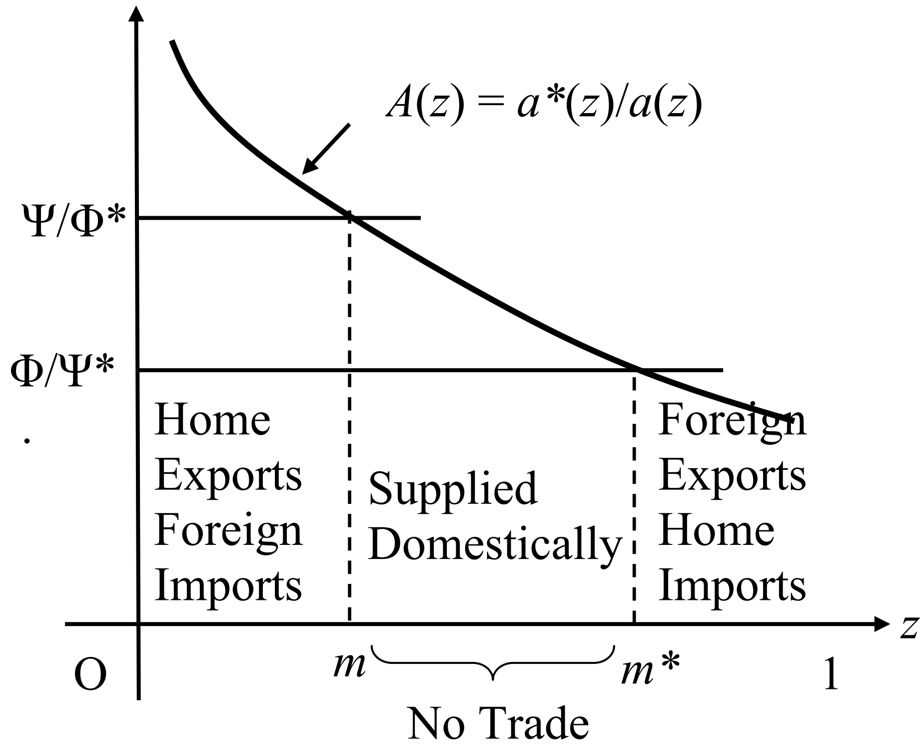
Patterns of Trade:

$$p(z) = \begin{cases} a(z)\Phi(w) & \text{if } z < m^* \\ a^*(z)\Psi^*(w^*;\tau^*) & \text{if } z > m^* \end{cases}$$

$$p^*(z) = \begin{cases} a(z)\Psi(w;\tau) & \text{if } z < m \\ a^*(z)\Phi^*(w^*) & \text{if } z > m \end{cases}$$

where

$$A(m) = \frac{\Psi(w;\tau)}{\Phi^*(w^*)}; \; A(m^*) = \frac{\Phi(w)}{\Psi^*(w^*;\tau^*)}.$$



- (A2) implies $m < m^*$, hence there are (endogenously determined) nontraded goods.
- The gap, $\dfrac{A(m)}{A(m^*)} = \dfrac{\Psi(w;\tau)}{\Phi(w)}\dfrac{\Psi(w^*;\tau^*)}{\Phi(w^*)}$, which may be viewed as the trade cost, depends on the factor prices. In DFS (1977), it is a constant parameter.

Calculating the Factor Demands:

Demand for Home Factor-j by the Home Domestic Sector $z \in [0, m^*]$*:*

$$a(z)\Phi_j(w)D(z) = p(z)\frac{\Phi_j(w)}{\Phi(w)}D(z) = \frac{\Phi_j(w)}{\Phi(w)}b(z)wV\,,$$

By integrating over [0, m*],

Domestic Demand for Home Factor-j: $\dfrac{\Phi_j(w)}{\Phi(w)}B(m^*)wV$

Demand for Home Factor-j by the Home Export Sector $z \in [0, m]$*:*

$$a(z)\Psi_j(w;\tau)D^*(z) = p^*(z)\frac{\Psi_j(w;\tau)}{\Psi(w;\tau)}D^*(z) = \frac{\Psi_j(w;\tau)}{\Psi(w;\tau)}b^*(z)w^*V^*$$

By integrating over [0,m],

Foreign Demand for Home Factor-j: $\dfrac{\Psi_j(w;\tau)}{\Psi(w;\tau)}B^*(m)w^*V^*$

Total Demand for the Home Factor-j:

$$V_j = \frac{\Phi_j(w)}{\Phi(w)}B(m^*)wV + \frac{\Psi_j(w;\tau)}{\Psi(w;\tau)}B^*(m)w^*V^*$$

By multiplying by w_j,

Home Factor-j Market Equilibirum:

$$w_j V_j = \alpha_j(w) B(m^*) wV + \beta_j(w;\tau) B^*(m) w^* V^*,$$

$\alpha_j(w) \equiv \dfrac{w_j \Phi_j(w)}{\Phi(w)}$; factor-j's share in Home domestic sectors, with $\sum_{j=1}^{J} \alpha_j(w) = 1$,

$\beta_j(w;\tau) \equiv \dfrac{w_j \Psi_j(w;\tau)}{\Psi(w;\tau)}$; factor-j's share in Home export sectors, with $\sum_{j=1}^{J} \beta_j(w;\tau) = 1$.

Similarly,

Foreign Factor-j Market Equilibrium:

$$w_j^* V_j^* = \alpha_j^*(w^*)[1 - B^*(m)] w^* V^* + \beta_j^*(w^*;\tau^*)[1 - B(m^*)] wV,$$

$\alpha_j^*(w) \equiv \dfrac{w_j^* \Phi_j^*(w^*)}{\Phi^*(w^*)}$; factor-j's share in Foreign domestic sectors, with $\sum_{j=1}^{J} \alpha_j^*(w^*) = 1$

$\beta_j^*(w^*;\tau^*) \equiv \dfrac{w_j^* \Psi_j^*(w^*;\tau^*)}{\Psi^*(w^*;\tau^*)}$; factor-j's share in Foreign export sectors, $\sum_{j=1}^{J} \beta_j^*(w^*;\tau^*) = 1$

Equilibrium conditions:

Patters of Trade:

(1) $A(m) = \dfrac{\Psi(w;\tau)}{\Phi^*(w^*)}$; (2) $A(m^*) = \dfrac{\Phi(w)}{\Psi^*(w^*;\tau^*)}$

Factor Market Equilibrium:

(3) $w_j V_j = \alpha_j(w) B(m^*) wV + \beta_j(w;\tau) B^*(m) w^* V^*$ (j = 1, 2,…J)

(4) $w_j^* V_j^* = \alpha_j^*(w^*)[1 - B^*(m)] w^* V^* + \beta_j^*(w^*;\tau^*)[1 - B(m^*)] wV$ (j = 1, 2,…J)

Adding up (3) or (4) for all j yields

Balanced Trade Condition:

(5) $B^*(m) w^* V^* = [1 - B(m^*)] wV$

These 2J+1 conditions determine 2J+1 unknowns (m, m*, 2J −1 relative factor prices).

Unbiased Globalization: Restoring DFS (1977).

(A3) $\Psi(w;\tau) = \tau\Phi(w) > \Phi(w)$; $\Psi^*(w^*;\tau^*) = \tau^*\Phi^*(w^*) > \Phi^*(w^*)$

- No Factor Intensity Differences: $\beta_j(w;\tau) = \alpha_j(w)$; $\beta_j^*(w^*;\tau^*) = \alpha_j^*(w^*)$
- Hicks-Neutral Export Technology Improvement (a shift in τ and τ*)
- τ – 1, τ* – 1; the trade barriers imposed by the trading partner

Equilibrium:

(5) $B^*(m)w^*V^* = [1 - B(m^*)]wV$

(6) $A(m) = \dfrac{\tau\Phi(w)}{\Phi^*(w^*)}$;

(7) $A(m^*) = \dfrac{\Phi(w)}{\tau^*\Phi^*(w^*)}$

(8) $w_j V_j = \alpha_j(w) wV$ (j = 1, 2,…J)

(9) $w_j^* V_j^* = \alpha_j^*(w^*) w^* V^*$ (j = 1, 2,…J)

Aggregation Theorem:

Define $F(x) \equiv \min_q \{qx \mid \Phi(q) \geq 1\}$. Then,

(10) $wV = \Phi(w)F(V) = WL$

$W \equiv \Phi(w)$: the Home factor price index, "the Home wage rate"
$L \equiv F(V)$: the Home factor quantity index, called, "Home labor."

Define $F^*(x) \equiv \min_q \{qx \mid \Phi^*(q) \geq 1\}$,

(11) $w^*V^* = \Phi^*(w^*)F^*(V^*) = W^*L^*$,

$W^* = \Phi^*(w^*)$: "Foreign wage rate"
$L^* = F^*(V^*)$: "Foreign labor"

Equilibrium:

$$\begin{cases} (12)\ \dfrac{A(m)}{\tau} = \dfrac{B^*(m)L^*}{[1-B(m^*)]L} \\ (13)\ A(m^*)\tau^* = \dfrac{B^*(m)L^*}{[1-B(m^*)]L} \end{cases}$$

$$\begin{cases} (14)\ V_j = \Phi_j(w)L & (j = 1, 2,\ldots J) \\ (15)\ V_j^* = \Phi_j^*(w^*)L^* & (j = 1, 2,\ldots J). \end{cases}$$

Eqs. (12)-(13) determine m and m*,
as a function of τ and τ*, independent of w, w*, V and V*.

Eqs. (14)-(15) determine w and w*,
as a function of V and V*, independent of m, m*, τ and τ*.

The model is Ricardian under (A3), i.e., without factor intensity differences between domestic and export sectors.

Biased Globalization:

We make the mirror image assumption (M):

(M) $A(z)A(1-z) = 1$; $\Phi = \Phi^*$, $\Psi = \Psi^*$, $\tau = \tau^*$; $B(z) = B^*(z)$, $B(z) + B(1-z) = 1$ for $z \in [0,1/2]$; $V = V^*$.

$A(z)A(1-z) = 1 \rightarrow A(z) > 1$ for $z \in [0,1/2)$; $A(1/2) = 1$; $A(z) < 1$ for $z \in (1/2, 1]$

Equilibrium: Symmetric, $w = w^*$, $m = 1-m^* < ½$,

(16) $A(m) = \dfrac{\Psi(w;\tau)}{\Phi^*(w^*)}$

(17) $w_j V_j = \{\alpha_j(w) + [\beta_j(w;\tau) - \alpha_j(w)]B(m)\}wV$ (j = 1, 2,…J)

An improvement in export technologies (a shift in τ) can affect factor demands in TWO separate routes: **Composition and Direct Effects**

Composition Effect: *By Shifting the Activities from the Domestic to Export Sectors*

A higher m or B(m) affects factor demands due to factor intensity differences between the Domestic and Export Sectors ($\alpha_j \neq \beta_j$)

Direct Effect: *By Changing the Factor Intensity of the Export Sectors*

$\beta_j(w;\tau)$ depends on τ.

We focus on the Composition Effect by assuming Hicks-neutrality.

(A4) $\Psi(w;\tau) = \tau\Psi(w)$ with $\tau > 1$ and $\Psi(w) > \Phi(w)$.

$\beta_j(w;\tau) = \beta_j(w)$

Again, τ can be interpreted as the trade barriers.

Equilibrium

(16) $$A(m) = \frac{\Psi(w;\tau)}{\Phi^*(w^*)}$$

(17') $w_j V_j = \{\alpha_j(w) + [\beta_j(w) - \alpha_j(w)]B(m)\} wV$ (j = 1, 2,…J)

Two-Factor Case (J = 2)

(18) $$A(m) = \frac{\psi(\omega;\tau)}{\varphi(\omega)}$$

(19) $$\omega\frac{V_1}{V_2} = \frac{\alpha_1(\omega) + [\beta_1(\omega;\tau) - \alpha_1(\omega)]B(m)}{1 - \alpha_1(\omega) - [\beta_1(\omega;\tau) - \alpha_1(\omega)]B(m)},$$

where $\omega \equiv w_1/w_2$ $(= \omega^* \equiv w_1^*/w_2^*)$, $\varphi(\omega) \equiv \Phi(\omega,1) = \Phi(w_1,w_2)/w_2$, $\psi(\omega;\tau) \equiv \Psi(\omega,1;\tau) = \Psi(w_1,w_2;\tau)/w_2$; $\alpha_1(\omega) = 1 - \alpha_2(\omega)$; $\beta_1(\omega;\tau) = 1 - \beta_2(\omega;\tau)$

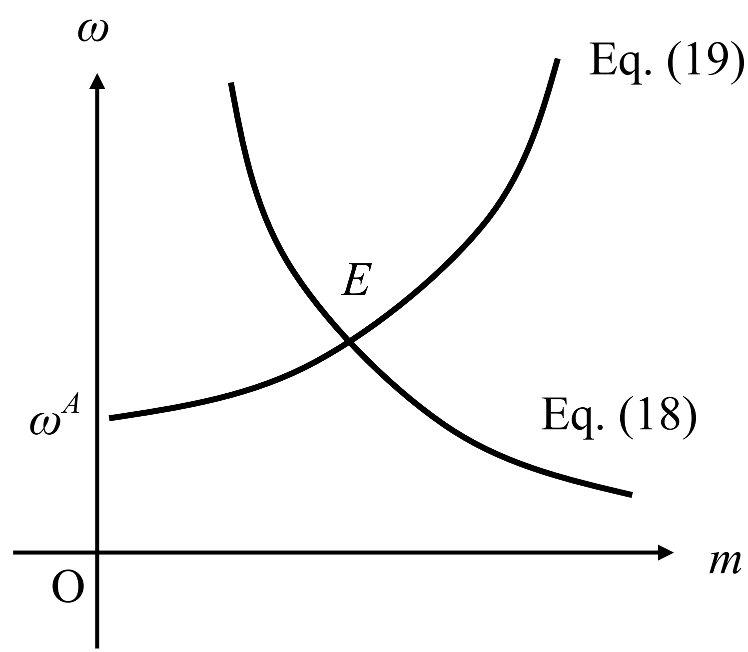
Factor Intensity Assumption: $\alpha_1(\omega) < \beta_1(\omega;\tau)$.

Eq. (18) is downward-sloping.

A lower ω
→ The cost of the export sectors declines relative to the domestic sectors
→ A higher m and A lower $m^* = 1-m$.

Eq. (19) is upward-sloping.

A higher m and a lower $m^* = 1-m$
→ A higher relative demand for factor 1

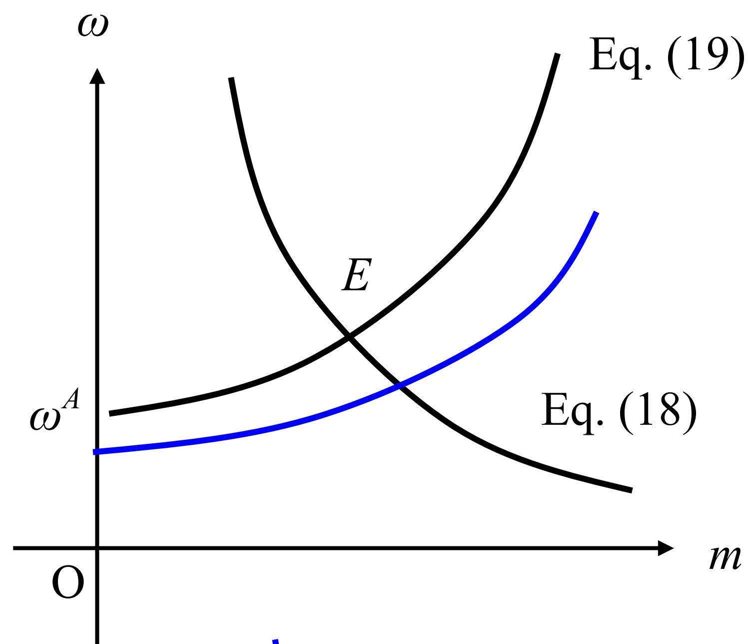


Some Comparative Statics:

A higher V_1/V_2: → A higher m and a lower ω.

A *world-wide* increase in the relative supply of the factor used more intensively in export activities leads to globalization.

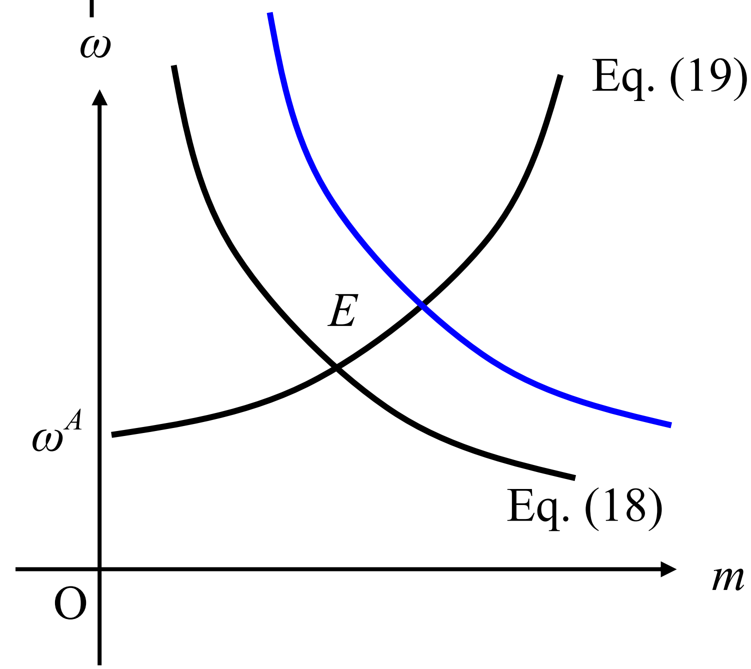
A Contrast to the Heckscher-Ohlin Mechanism



A Decline in τ → A higher and a higher ω.

An improvement in the export technologies (or a decline in the trade barriers) leads to globalization and a *world-wide* rise in the relative price of the factor used intensively in the export sectors.

A Contrast to the Stolper-Samuelson Mechanism



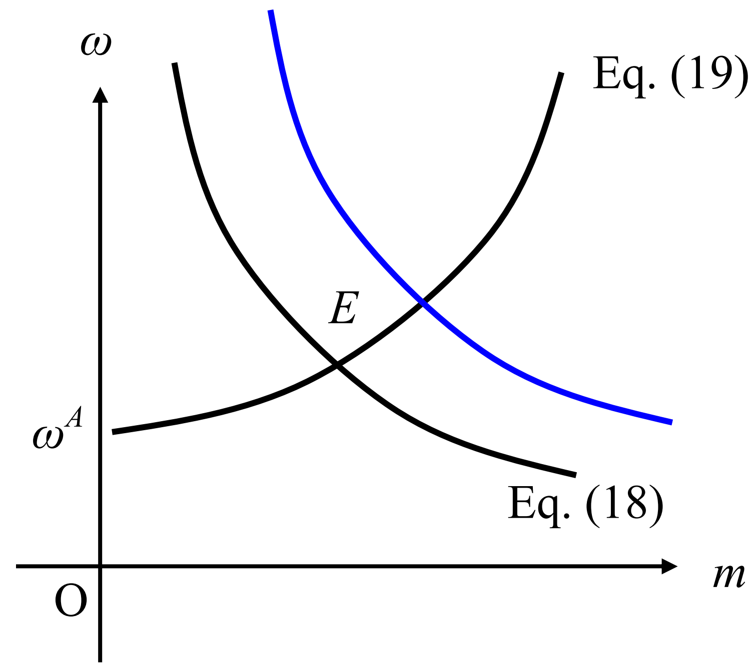
A change in A(z).

Generalize A(z) to $[A(z)]^{\theta}$, with $\theta > 0$.

A higher θ magnifies the TFP difference

More Reasons to Trade

→ An increase in both m and ω.



An Application: Globalization, Technical Change, and Skill Premia

Two Approaches

1st Approach: Globalization *is* inherently skilled-biased

Assume that Export Sector is More Skill Intensive than Domestic Sector

- A world-wide increase in the relative supply of skilled labor leads to globalization
- An improvement in the export technologies or a reduction in the trade barriers leads to globalization and a world wide rise in the skill premia.

2nd Approach: Globalization *induced by* skilled-labor augmenting technical changes.

Two-Factors; Skilled and Unskilled Labor

Unit Cost Functions		Destination	
		Home	Foreign
Origin	Home	$a(z)\Phi(\tau w_s, w_u)$	$a(z)\Psi(\tau w_s, w_u)$
	Foreign	$a^*(z)\Psi^*(\tau^* w_s, w_u)$	$a^*(z)\Phi^*(\tau^* w_s^*, w_u^*)$

A reduction in τ (and τ^*) means a skilled-labor augmenting technical change, and hence it reduces the costs of both the domestic and export sectors for fixed wage rates.

(A5) $\Phi(\tau w_s, w_u) < \Psi(\tau w_s, w_u)$ and $\Phi^*(\tau^* w_s^*, w_u^*) < \Psi^*(\tau^* w_s^*, w_u^*)$.

Under the Mirror Image Assumption,

(20) $$A(m) = \frac{\psi(\tau\omega)}{\varphi(\tau\omega)}$$

(21) $$(\tau\omega)\frac{V_s/\tau}{V_u} = \frac{\alpha_s(\tau\omega) + [\beta_s(\tau\omega) - \alpha_s(\tau\omega)]B(m)}{1 - \alpha_s(\tau\omega) - [\beta_s(\tau\omega) - \alpha_s(\tau\omega)]B(m)},$$

Skill Premia: $\omega \equiv w_s/w_u$ (= $\omega^* \equiv w_s^*/w_u^*$)

Under the Hicks-Neutrality Assumption (Uzawa's Theorem)

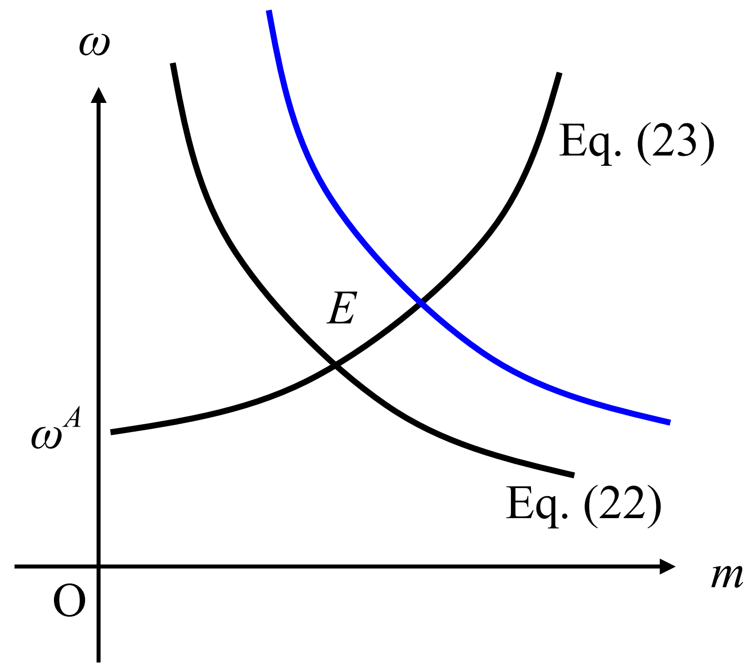
(A6) $\Phi(\tau w_s,w_u) = (\tau w_s)^{\alpha}(w_u)^{1-\alpha}$, $\Psi(\tau w_s,w_u) = \Gamma(\tau w_s)^{\beta}(w_u)^{1-\beta}$

with Γ large enough to ensure that $\Phi(\tau w_s,w_u) < \Psi(\tau w_s,w_u)$ in equilibrium.

If Export Sector is More Skill Intensive: $0 < \alpha < \beta < 1$.

(22) $A(m) = \Gamma(\tau\omega)^{\beta-\alpha}$,

(23) $\omega \dfrac{V_s}{V_u} = \dfrac{\alpha + (\beta-\alpha)B(m)}{(1-\alpha)-(\beta-\alpha)B(m)}$,



A skilled-labor augmenting technical change (a reduction in τ) shifts the downward-sloping curve to the right, leading to globalization and an increase in skill premia.

Beyond Icebergs in a HO Model with a Continuum of Goods (Unfinished)

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